About the “true” power coefficient between the curvature and velocity – and what does it have to do with PCT

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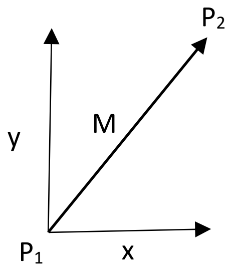
This brief composition is a trial to one more time express as simply and understandably as possible the criticism against the central claims of Marken & Schaffer 2017. The ideas of this writing are mostly already expressed in Zago et al 2018 and Taylor 2018 and in many discussion messages with broader justifications and more details. I will try to concentrate only into the most important key topics. The main argument in Marken & Schaffer is multipart and not easy to interpret. Its main content and motivation seem to be the claim that because the power law phenomenon is an irrelevant side effect of control its study is a waste of time. (At least this idea repeats in discussion messages of Marken.) This side effect claim is supported by the claim that the power law is a statistical artefact created by the researchers. This claim is then supported by the most staggering claim that there is a necessary mathematical dependency between the curvature and velocity of (any) movement so that the 1/3 power law prevails generally and exactly – despite the mass of different empirical cases.

1. About the fundamental independence of dimensions

Let’s consider a simple planar movement in a two-dimensional system of coordinates, like this:



The vector M describes a movement from the point P_1 to the point P_2. This kind of movement is described – for example in PCT – as having two degrees of freedom. This means that the x and y components of the movement, described here as x and y vectors (or rather their lengths), can change independently of each other. There is no mathematical relationship between them or mathematical law which says that if x has *this* certain value, then y must have *that* certain value. No such relationship or law does exist.

But of course, there is a relationship between M, x, and y after the movement has taken place: As vectors they obey the relationship M = x + y and as lengths $M = \sqrt{x^2 + y^2}$. These relationships mean that if we happen to know M or something about it, we can infer something about x and y; and if we happen to know the values of two of the relata, we can calculate the value of the third. But these relationships are conditional in the sense that we can do these inferences and calculations only after the movement has taken place and M has a definite value. These relationships do not beforehand violate the absolute freedom of x and y to vary independently of each other.

The vectors x and y do not only determine the length of M but also at same time its direction. Because x and y can change independently, then also M’s length and its direction can change independently. From pure length we cannot infer direction and vice versa (if we do not happen to know the length of x or y).

Adding time into the consideration does not change the situation. If the movement took t amount of time, then its velocity (or speed) is M/t. Similarly, we will have x and y components of velocity: x/t and y/t. The velocities of x and y are as independent as are their lengths.

From that basic independence of x and y follows also that any two pieces of movement, say M_1 and M_2 are mathematically independent. The length or the direction or the velocity of M_1 can no way (mathematically) determine any of those of M_2. Not even if they happen to be consecutive steps of the continuous movement of one and the same object. There is no mathematical relationship or law which says that if one step of the movement was *this* kind, then the next must be *that* kind. If there appears to be mathematically expressible dependency relationships between any components of any movement, then these dependencies must be caused by something else than pure mathematics. The causes of these apparent dependencies can be at least on the one hand the different physical effects which affect the production of the movement or on the other hand the methods of observing, measuring, and calculating the features of the movement – or both.

2. About conditional relationships

As said above, we can create conditional relationships between two independent variables by defining a third variable by combing the two original variables. One example is the mentioned Pythagorean theorem between the lengths of the movement and its x and y components, but it is not all the only one or only kind. We can also define for example:

$n = x + y$ or

$n = x * y$ or

$n = 2 * x^{1/7} + y^5 / 3$

or any. The value of n need not be especially useful but still these definitions create a conditional relationship between x, y, and n so that we can calculate the value of the third variable if we happen to know the values of the two others. Some definitions can be useful like the area of a rectangular or affine velocity of the movement, but they are useful only for certain uses. Those others could be useful also if you invent a suitable use, but still, they all similarly create the conditional relationship.

If we mark the curvature (the amount of the change of direction of the movement at certain point) as C and the velocity (at the same point) as V, we can define affine velocity (V_A) (for example) as:

$V_A = C^{1/3} * V$ (from Pollick & Sapiro)

Affine velocity is a useful concept because there prevails a handy conditional dependence so that **if** we know that the value of affine velocity remains constant in some movement, **then** we can infer and predict that the relationship between velocity and curvature obeys 1/3 power law. But if affine velocity refuses to stay constant then we cannot infer much anything. This conditional dependency does not at all explain the power law phenomenon but just changes the question to “why affine velocity sometimes (and quite often, but not at all always) remains constant?”

Then we could define another concept, say *beffine velocity* (V_B), like this:

$V_B = C^{1/4} * V$

I am not sure what we could infer from the values of V_B (perhaps the constant V_B means ¼ power law – who wants to test?), but still, it is mathematically as valid concept as V_A and similarly creates a conditional relationship between C, V, and V_B.

As Matic and Gomez-Marin has just shown, angular speed (marked as A) can be seen as quite similar construct as those above:

A = C * V

They state that the angular speed should not be used instead of the tangential speed (or velocity as I have called it here) because it causes over-estimations of the correlations between curvature and velocity. It can produce power law phenomena to those cases where there originally are none. It does not affect the power coefficient but only unrealistically strengthens the correlations – so it creates statistical artifacts. Why does it do this? Because the analyzed correlation between C and A is the same as correlation between C and C * V. In other words, it also contains the correlation between C and C, in addition to that between C and V.

3. About the "Omitted Variable Bias"

One of the first things I was taught in the statistical methods course a long time ago was that one should not try to calculate correlations between conceptually dependent variables for two reasons: First, it is needless because the dependency can be inferred from the meanings or definitions of these variables. An example is sex and bachelorhood in the human population. Secondly, the aim of scientific research is to find explanations for phenomena for which we do not yet have satisfying explanations. For conceptual dependencies we always already know the explanations (= the definitions). But for many physically caused dependencies we do not know, and it is just them that we are searching for in science. Now if you mix accidentally conceptual and physical dependencies – like in the case of using angular velocity in power law research – you will distort the results and probably hide something important.

Marken and Schaffer claims that the "true" power coefficient between curvature and velocity can be unveiled by multiple correlation analysis where affine velocity (or actually in the paper they used a "cross product" $D = V^3 * C$) is added to the predictors in addition to curvature. The nonlinear power relationship can be inferred from the linear dependency between the logarithms of the variables. So normally the correlation between log(C) and log(V) is analyzed like this:

log(C) ----> log(V)

But if we instead analyze:

log(C); AND log(C) + 3 * log(V) ----> log(V)

we will be doing just what Matic and Gomez-Marin warns about: adding a correlation of velocity with itself to the result, and in addition to that, because of the form (the multipliers) of the definition, we also at the same time distort the power efficient.

So, the core problem is not at all the concept of affine velocity (or cross product D) as such and how is it derived, but only how it is used. Adding it (or any other similar compound variable, like that *beffine velocity*) to the correlation / regression analysis between curvature and velocity is a cardinal crime against sound statistical methods and will create fake results, a statistical artifact.

4. About side effects

In a recent message Marken now clarifies that *irrelevant side effect of control* means an unintended consequence of control that is "irrelevant to the controlling done by a control system". That is of course quite true about power law phenomenon. There is a broad consensus at among PCT researchers that power law cannot be a controlled variable simply because it is extremely difficult to perceive the power relation between velocity and curvature without special scientific measurement methods, and if you cannot perceive something then you cannot control it neither – and it is irrelevant to you.

Earlier many times Marken claimed that power law as a side effect of control does not tell us anything about how the movement is produced – and that is why it would be futile to research it scientifically. (Power law research was to him an example of behavioral illusion.) Probably it was just to defend this claim, why he armed himself with those irrational arguments about necessary mathematical relationship between velocity and curvature and 1/3 as its "true" power coefficient. If that argument were valid, then all possible movements along all possible curvature profiles and with all possible velocity profiles would obey 1/3 power law – no matter are they caused by living subjects, machines, or planet systems. Then of course power law would not tell anything about how the movement is produced – but neither were there any reason to call it a side effect to control.

Marken also claimed many times that power law is a statistical artifact produced by the researchers. If this were the case then it could neither be a side effect of the control of the movement producing system, but rather an effect of the controlling done by the researchers.

It is a general PCT mantra: "Many Means to the same End". This means that controlling of some variable can utilize many different lower-level controlling systems and finally many different output profiles in the environment. The output in the environment causes in addition to the main effect which affects the controlled perception also many kinds of more or less irrelevant side effects. These side effects are not caused (only) by the environment but like all effects they require interaction and thus side effects always tell something about how the output was produced. Different output causes different side effects. This is clearly visible in the empirical results that different curvature and velocity profiles have different power relations.

Two examples of relevance of power law: 1. If the subject controls for moving along an elliptical trajectory with a stable velocity, it will succeed in low velocity but fail in higher velocity. Interestingly the subject cannot necessarily perceive this failure because the power law velocity is seen as stable velocity. But if for some reason it were important to remain stable velocity and if the subject could see the exact velocity from a meter instrument, then there would be problem in the control. 2. In medical research it has been shown that 1/3 power law movement (= stable affine velocity) predicts skillfulness of the hand movements in some difficult surgical operations.

5. Conclusion

Marken has many times criticized what he calls "theory first" approach. With this he means the way to do research so that the features of theory are studied. Instead, he suggests "data first" or "phenomena first" approach, where only the features of some data set are analyzed first and only after that any theoretical conclusions are made. However, in this discussion Marken has had a strong theoretical starting point according to which power law is an irrelevant side effect of control. He has proved to be ready to defend this "truth" by making irrational arguments and even by ignoring and twisting the empirical data.

(Thanks to Adam and Martin for correcting a couple of errors.)