

Modelling Systems with Intentional Dynamics:
A Lesson from Quantum Mechanics

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"Perception is virtual action" (Henri Bergson)

"Action is virtual perception" (The Authors)

1.0 Introduction: The Goalpath Navigation Problem

For ecological psychology it has been suggested that the perceiving-acting cycle should be the smallest unit of analysis. We would like to amend this suggestion. It now seems to us that the smallest unit of analysis must be *the perceiving-acting cycle situated in an intentional context*. What this means is the main topic of this paper.

To situate the perceiving-acting cycle under intentional constraints is to identify a space-time context in which the actor selects a goal¹, and then selects from all causally possible paths those that are potential goalpaths, and from these the actual goalpath to follow. The generic problem, therefore, is how best to describe *the action* of an organism. A successful *action* (henceforth defined as *a goal-directed behavior*) minimally entails selecting a (distal) target and moving over a space-time path in an intended manner to that target. This implies that the distal target and the future goal-state of the actor must make their presence felt in the information and control accompanying its current state. Thus somehow the distal must logically condition the proximal, so that the actor's changing relationship to the intended final condition acts to re-initialize (update) the actor's current condition. This is what it means for a space-time path to be *a goalpath*.

A careful consideration of these requirements suggests that *a field of control-specific information* must exist in which the actor and the intended goal both participate. Furthermore, this field of control-specific information must at the same time and in the same place be *a field of goal-relevant control*. Hence each space-time locale in the field is characterized by both an information value and a control value. Such values that go together in this dual fashion are said to be *conjugate information-control values*. The relationship between the energy controlled and the information

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¹ We take *goalpath* knowledge to comprise both target information and manner determination—that is, specification of where the target is located in space-time and determination of the best manner of approaching it. (See Shaw & Kinsella-Shaw, 1988, for details)

detected with respect to the goal is said to be *adjoint* by nature—something all creatures are born into because of evolutionary design. When this adjoint relationship, however, leads to successful goal-directed behavior (something that often has to be learned), then the adjoint relation of information to control is said to have become *self-adjoint*.²

One recognizes in this problem of goal-directedness the need for what the Würtzburger ('imageless thought') School of psychology called a 'determining or organizing tendency' (*Einstellung*). We call this *Einstellung*, when augmented with a boundary condition, an *intention* because it is the goal-sensitive, agentive function (a *cognitive operator*, if you will) which determines goal-selection and organizes the dynamics under a 'control law' designed to serve the intention.³

The existence of such a field of conjugate values, in which information and control might become self-adjoint, would explain how everywhere that the animal might venture there are opportunities for acting toward the goal in an intended manner (excluding, of course, those places and times where the target is occluded or barriers block its accessibility). *We shall show that such an information-control field has a natural interpretation in an adjoint information/control theoretic formulation of quantum mechanics.*

As a step toward this field theoretic model, we postulate an *intentional process* which acts (as an *Einstellung*) to set up a perceiving-acting cycle (along the lines discussed by Kugler & Turvey, 1987, and Shaw & Kinsella-Shaw, 1988). The actions that the perceiving-acting cycle *might* generate over space-time define the causally possible family of goalpaths. Here intention, defined as a cognitive operator, tunes the perceiving-acting cycle by directing both the attention and the behavior of the actor toward the goal. A coherent account of this intention-driven dynamics would remove the mystery of how actors maintain informational contact with their goalpaths; namely, they do so by direct perception when the goal is detectable, or otherwise, when not detectable (say, over the horizon), they must navigate either by *indirect perception* or by direct perception *plus dead reckoning*. For humans, indirect perception may be achieved, as Gibson (1979) suggests, by means of verbal instructions, or by use of a map (with target coordinates specified), perhaps, drawn or remembered. As nautical navigators discovered, however, a map alone is not adequate; one also needs a compass to determine directions at choice-points, and a chronometer to satisfy a schedule of departure and arrival times if contact with the goalpath course is to be maintained.

² In this paper we will discuss the role of adjointness and self-adjointness in perceiving and acting in an intentional context. We will not discuss, however, the nonlinear learning or 'insight' process by which adjointness becomes self-adjointness. That will have to await further developments of the theory of intentional dynamics. But see Shaw & Alley, 1977, and Shaw, Kadar, Sim, & Repperger, 1992, for initial thoughts on learning.

³ By relating intentional dynamics to this central concept of the Würtzburger school of act psychology, we underscore Gibson's (1979) acknowledgment that ecological psychology has a close historical tie to this school because both oppose the elementarism of the structuralists and emphasize process.

Hence the approach proposed in this paper can be summarized as a *field of conjugate information-control values, with paths being generated by a perceiving-acting cycle which is motivated and guided by intention as a field process*. This account contrasts sharply with more traditional accounts. Let's consider the contrast:

Since animals presumably do not use navigation tools, then they (like humans without benefit of maps, compass, and clocks) must rely on direct perception *plus dead reckoning* to perform the same navigation functions. Traditional psychology assumes, not unreasonably, that under such circumstances they direct themselves by 'cognitive maps' (where intended goals are somehow attentionally distinguished from non-goals). The evidence for the existence of cognitive maps, one might argue, is the actor exhibiting a 'sense of direction' at choice-points, and a 'sense of timing' which keeps the actor on schedule in arriving at and departing from sub-goals. Here the cognitive modelling strategy proceeds by positing internal mechanisms that internalize the map, compass, and chronometer functions. Regardless of either the truth or usefulness of such internal constructs, the success of the internal state modelling strategy is predicated on a successful actor's having access to goal-specific information and goal-relevant control along the goalpath. The field notion also putatively captures the sense of the *social invariance* of the information and control opportunities which

- (a) allows an observer to see which goal an organism is most likely pursuing, and
- (b) allows different organisms to compete for the same goal.

Hence one may debate whether the field of information and control manifests itself *internally* (as cognitive psychologists maintain), *externally* (as behaviorists have maintained), or *dually* (as we ecological psychologists propose), but the field's existence is without question, being assumed by all parties alike. (See Shaw & Todd, 1980; Shaw & Mingolla, 1981; and Shaw, Kugler, & Kinsella-Shaw, 1991, for a comparative description of these alternative approaches).

Regardless of whether navigation is achieved by direct or indirect perception, the actor's control process must maintain invariant contact with the intended goal over some dynamically developing course of action—a potential goalpath. Consequently, a theory is needed for what constitutes goalpaths, and how they are recognized, selected, and followed. We assume that a goalpath is *generated*, as a segment of a worldline in space-time, by the actions of the perceiving-acting cycle engaged by the organism. Before considering the details of how this engagement is to be formally characterized, we consider the general intuitions that underwrite the intentional dynamics approach to this problem.

1.1 Modelling Systems that Exhibit Intentional Dynamics

Intentional dynamics *inter alia* faces two problems:

First, how is the perceiving-acting cycle comprising a dual relationship between information and control to be formally described? In Section 2 an answer is proposed from the perspective of a variant on optimal control theory called *adjoint systems theory* (Shaw, Kadar, Sim, & Reppeger, 1992).

Second, how is the field of conjugate information and control values available to the perceiving-acting cycle to be made formally explicit? Here we borrow from quantum mechanics the image of a particle being involved in measurements as it moves through a field toward an attractor.⁴

The goal of Section 3 is to provide the generic mathematical description of an organism with a complex interior, being driven by internally produced forces and guided by externally available information onto a goalpath toward a future goal-state. This image of a complex animate 'particle' exhibiting intentional dynamics in a field of information and control replaces the standard image of a particle with a simple interior, being driven by outside forces onto a 'least action' path that is indifferent to any future goal state.

Given an actor at some space-time location who intends to connect with an accessible target at some other space-time location, then there will exist a family of causally possible goal-paths. This set is bounded in space-time by the maximum rates of causal action allowed by the (e.g., locomotory) capabilities of the agent who intends the goal. For convenience, we call such a bounded region of goalpath possibilities, an Ω -(omega) cell—a construct of *ecological physics* which falls between the cosmological scale and the quantum scale (Shaw & Kinsella-Shaw, 1988). At each moment, along each path there is a certain amount of energy the agent must control if the action is to be in the goal's direction. The amount of control is perceptually specified at each of these points on each goalpath by goal-specific information. What form does this specification take?

This question poses, in part, a version of the so-called 'inverse dynamics' problem for psychology (Shaw, Kugler, & Kinsella-Shaw, 1991), whose solution has been discussed elsewhere (see Saltzman & Kelso, 1987; Shaw, Flascher, & Kadar, *in press*). But since the agent could be on any one of a number of paths, then some perspectively weighted information and

⁴ Strictly speaking there is no attractor as a minimum on a physical manifold located solely in either the environment or the organism (construed as exterior and interior fields, respectively) that can be the goal of the action, in the full meaning of the term. Rather the attractor dynamics governing goal-directedness must be defined on the *ecological quotient manifold* (exterior degrees of freedom/interior degrees of freedom) where the flow of generalized action is located (see Shaw, Flascher, & Kadar, *in press*). To our knowledge, there is no discussion of this fact in the literature where goal-directed behavior is attributed to dynamics on a physical attractor basin. The solution called for must be much more abstract. It must be defined over organism and environment, rather than either alone. Hence it must be an ecological physics manifold.

control quantity must be available at each point on each possible path. Quantum physics (as discussed in Section 3) offers us a lesson on how to do such weighting.

In Section 2, in preparation for the quantum field treatment, we show how, even in classical physics, a single quantity exists as an *inner (scalar) product* of information and control which is defined at each point along the goalpath (Shaw, Kadar, Sim, & Reppeger, 1992). We offer the following intuition as to what this means: From the internal frame of the actor, one might think of the control-specific information as a wave crest that accompanies the moving agent at each point along the goalpath—from initial to final condition. Let's call this a 'knowledge wave' since it embodies all the dynamical knowledge about the goal (namely, where it is and how to get there) available to the actor as an acting perceiver.

Alternatively, from an external frame of a scientific spectator, one might think of the 'knowledge wave', as it moves over the distribution of possible paths, as specifying at each point, on each path, the likelihood that a perceiving-actor, who intends the goal, will be found there. Hence intentional dynamics assumes that well-intentioned, normally competent actors will tend to go where goal-specific information is most likely to be found and goal-directed control is most likely to be achievable. Our aim in this paper is to show that the existence of such a 'knowledge wave' is by no means fanciful under the conception of intentional dynamics, as developed by us in earlier papers—although such a dynamical construct as a 'knowledge wave' has not before been introduced. Consequently, all the mathematics that follow are designed to explicate this intuitive interpretation. Our aim will be to show that *when the knowledge wave embodies information and control that are only adjoint with respect to the goal, then the actions taken can at most be relevant but unsuccessful. However, when they are self-adjoint, then the actions are, by definition, both relevant and successful.*⁵

Before mathematically developing this new explanatory construct, let's consider the current status of the theory of intentional dynamics that has emerged over the past five years or so. The purpose of the next section is twofold: To clarify what one might mean by the claim that *actions must be situated in an intentional context* and to give an overview of the problems that a theory of the intentional dynamics of such situated actions must face. We also indicate the extensions to the theory proposed by the current effort.

1.2 Intentional Dynamics: An Overview

In earlier work we proposed representing the perceiving-acting cycle of an actor as a continuous (Lie) group of complex involutions. This approach draws its inspiration and borrows its mathematical techniques from classical mechanics (e.g., Goldstein, 1980). The virtue of the

⁵ This notion of self-adjointness has been developed elsewhere under the guise of reciprocities of an intentional system (Shaw, Kugler, & Kinsella-Shaw, 1991).

continuous group representation is that it allows one to characterize the 'intentional' action of systems as the 'flow dynamics' of a *generalized Hamiltonian* action potential which follows paths dictated by a 'least action' principle. We have called this generalized approach *intentional dynamics* and attempted to clarify the notion of the new action potential as follows (Shaw, Kugler, & Kinsella-Shaw, 1991):

For a flow to exist [over a goalpath], there must be a force. A *force* can be defined as *the gradient of some potential*. In some sense a goal can be said to exert an *attractive force* on the system. The sense we suggest is as *some kind of potential difference* between the endpoints of a goal-path. For this to be more than mere metaphor, we must find some way of allowing the interior gradient of the organism's metabolic potential to interact with the exterior force field of the environment. This can only take place through the detection of perceptual information which, in turn, must guide the controllers of the neuro-muscular actuators. Hence the relevant potential difference, or *goal-gradient*, can only be defined over an interior (metabolic) potential relating the initial state of intending the goal to the final state of arriving at the goal. This gradient must also reflect the difference between a system's current manner of behavior *where it is* and the desired manner of behavior *where it wants to be*.

The trick is to get the interior gradient and the exterior gradient linearly superposed so that their resultant is the desired goal-gradient. But this raises another problem. What kind of strange potential is the goal-gradient to be defined over? What is this superposed potential that is neither solely energy nor solely information but both? We call the resulting potential a *generalized action potential*. Whatever this generalized potential is, it is what flows in bi-temporal directions, between the interior and exterior frames, over the perceiving-acting cycle. Furthermore, it is also what must be *conserved* under the intentional dynamics of any system when successfully seeking a goal (pp. 595-596).

In Shaw, Kugler, & Kinsella-Shaw (1991), we proposed a way that this 'trick' of superposition might take place. Furthermore, it was shown how such a generalized action potential might exist, as well as how such a quantity might be conserved (under the Liouville theorem) as a fundamental dynamical invariant of intentional systems.

On the other hand, this 'classical' approach failed to make clear how a particular goalpath is selected by the system from all causally possible goalpaths; rather we described mathematically only how the perceiving-acting cycle might move down *which ever* goalpath was selected. As in the original paper by Shaw & Kinsella-Shaw (1988), the 'extraordinary boundary conditions' posed on a dynamical system by the selection of a goal are not defined, only assumed. In the present paper we seek to remedy this problem. Here we offer an explicit mathematical description

of how an actor's intention to pursue a goal automatically does two things: *First, the intention to act imposes the 'extraordinary' information and control boundary conditions on the action taken, and, second, the action selects if not the actual goalpath, then: the most probable one to be followed.*

Furthermore, we need to show how getting the appropriate mathematical description of the generalized action potential assigns a probability value to each path in the distribution of potential goalpaths. The probability value provides a *likelihood estimate* of the path being selected by the perceiving-acting cycle as the 'best' route to the goal, given the confluence of environmental and biomechanical constraints. 'Best' here means the practicable compromise between the mathematically ideal and the physically achievable, what can be thought of as the *tolerably suboptimal path* (Shaw, Flascher, & Kadar, *in press*). But how are information and control to be coupled to form a perceiving-acting cycle that can select such a goalpath?

Between the moment of the intent to pursue a goal and the successful attainment of the goal, there exists a functionally defined, space-time region in which the intentional dynamics of the actor is well-defined. In four dimensional geometry any dynamical process is represented by an event which develops over a worldline segment. To understand intuitively the geometry in which goal-directed actions take place, one might first build a geometry for events (Shaw, Flascher, & Mace, *in press*). For example, Figure 1 shows the standard light cone from the Minkowski rendition of special relativity. (Here the third spatial dimension is omitted). The backward temporal cone, called the *domain of (causal) influence*, indicates all those events ⁶ in the past that might causally affect the event at the origin (vertex). By contrast, the forward temporal cone, called the *domain of (causal) dependence*, indicates all those events in the future that might be affected by the event at the origin.

(Insert: Figure 1: A Minkowski Light Cone)

The standard light cone is not adequate for depicting goal-directed behaviors since its worldlines are unbounded. Instead, we need a new four-dimensional geometry in which the worldlines representing goal-directed actions are bounded by endpoints. Figure 2 depicts this new geometry. Imagine, for sake of illustration, that you are given the task of spinning a turntable manually through four successive half-turns ($4 \times 180^\circ$). The kinematics of this goal-directed action is shown below.

⁶ In space-time there are point-events and worldline paths for ongoing processes. Here we considered events to be finitely bounded segments of worldlines.

(Insert: Figure 2: An Ω -cell Geometry for a Goal-directed Action)

The Ω -cell's four dimensional geometry is a *lattice* structure, and therefore has three possible partitions (see Figures 2 and 3): the maximum partition, or *least upper bound*, noted by Ω -cell = ω_{max} , the minimum partition, or *greatest lower bound*, noted by α -cell = ω_{min} , and the intermediate partitions, noted simply by ω -cell = ω . Thus, in general, any form of goal-directed behavior will have a lattice structure within the geometry of the Ω -cell as indicated by

$$\Omega\text{-cell} = \omega_{max} \geq \omega \geq \omega_{min} = \alpha\text{-cell}.$$

In the turntable task, the Ω -cell partition corresponds to the overall intention of rotating the turntable through 720°; the ω -cell partition corresponds to the subgoals of rotating through two full rotations (2 x 360°); and the α -cell partition corresponds to the four 180° ballistic rotations (below which no choice-points are possible). A similar analysis generalizes to any goalpath with any number of partitions.

(Insert: Figure 3: A Schematic Ω -cell Showing its Nested Partitions)

An *econiche* for an organism (or species) is defined by *how* it lives in its habitat. *Affordances* present opportunities for action since they are possible goals. The character of an *econiche* is determined by its affordance structure. Indeed, an *econiche* is its affordance structure.

Effectivities correspond to the means required to carry out a control law by which an affordance goal is realized (what Gibson, 1979, referred to as a *rule for the perceptual control of action*). The repertoire of effectivities possessed by an organism determines what kind of actor it is. Indeed, an actor is its effectivity repertoire. In this sense, an ecosystem is the union of the affordance structure of an *econiche* and the effectivity system of an actor (or species of actors). A *situation* refers to *where* the relevant causal and informational constraints for an action exist. An *occasion* refers to *when* the need or value motivating the action is felt. An *effectivity* is brought to bear on an affordance goal when the actor *intends* to act so as to satisfy a motivating need or value.

All these ingredients (need or value, affordance goal, effectivity means, and intention, together with the implied forces to be controlled and information to be detected) must become a coherent unit of analysis if the intentional dynamics of an entailed action is to be understood. The theoretical construct under which all this comes together as an organized whole is, of course, *the Ω -cell*.

An organism's life as an actor is a 'tiling' of space-time by a concatenation of Ω -cells whose partitions parse the worldline of the actor from birth to death. Intentions are choices of affordance goals which functionally create the Ω -cells to be entered and hopefully crossed. The crossing requires the 'assembling' of an effectivity to engage, direct, and tune the appropriate perceiving-acting cycle to the exigencies of the task situation. The α -cell partitions of an Ω -cell represent the

tolerance limits on information detection and energy control—below which a kind of Heisenberg uncertainty is encountered. The ω -cell partitions designate those choice-points in control where a bifurcation set of possible paths exists. Here the actor, given an up-date on perceptual information, can alter the manner of approach to a sub-goal without abandoning the global goal defining the parent Ω -cell. These are the minimal constituents that must be captured in any theory of the intentional dynamics underwriting goal-directed actions. These intuitions are made formally explicit in Section 2 and 3.

Although functionally defined, Ω -cells have an objective reality. They determine the boundaries on behaviors which are tolerant of the same goal (i.e., target plus manner). Such nonlocal goal constraints have the same ontological status as forces in physics, for which evidence is also only functionally defined as a relationship between masses and their observed accelerations (direction and speed). The tolerance class of goalpaths (i.e., each being a velocity field) are parametric (manner) variants whose underlying invariant is their common goal-directedness. Where the *affordance goal* determines the final condition which constrains the resultant direction of the paths, the *effectivity* chosen determines which of the possible goalpaths within the Ω -cell is to be followed. Hence, in the case of a successful goal-directed behavior, an affordance goal—a functional property of the environment—is always complemented by an effectivity—a functional property of the actor. The intention, as a cognitive attunement operation, brings the necessary control and information to bear on the biomechanics of the actor. So long as the intention remains invariant, and *ceteris paribus*, the actor is perceptually guided down the goalpath.

Others have attempted to explain goal-directed behavior, but without the Ω -cell construct to consolidate the 'common fate' or 'determining tendency' of the variant but goal tolerant paths, little mathematical progress was possible (Ashby, 1952; 1956; Sommerhoff, 1950; Weir, 1984; Rosen, 1985). By building our theory of intentional dynamics around this fundamental concept, we show how the perceiving-acting cycle might be situated in an intentional context.

In the next section, we show how the perceiving-acting cycle can be modelled as a set of adjoint differential equations. Here the Ω -cell makes its appearance indirectly under the guise of a famous theorem regarding adjointness in control theory—the Kalman Duality theorem (Kalman, Englar, & Bucy, 1962). Finally, in Section 3, we show how the Feynman path integral approach (a version of quantum mechanics) can be combined with a generalized form of the Kalman Duality theorem. By doing so, we endeavor to obtain a complete and coherent account of the intentional dynamics by which a perceiving-actor 'knows' how to select the 'best' goalpath from among all possible alternatives.

2.0 The Classic Adjoint Systems Approach to the Perceiving-acting Cycle

Let us begin by anticipating what this section will show. A perceiving-acting system might be represented by a pair of dual differential equations—with one equation representing the system's control of energy and its adjoint equation representing its detecting of information. The pair of such equations are said to be *temporally dual* when *self-adjoint* because the original system exhibits a flow of time-forward control over the same space-time path that its adjoint system exhibits a counter-flow of time-backward information. Figure 4 portrays schematically the self-adjoint relationship proposed for information and control equations. Note how these quantities 'flow' in opposite temporal directions—with each endpoint doing double duty, serving as a repeller for one quantity and an attractor for the other quantity.

(Insert: Figure 4: Temporal Self-adjointness of the Information and Control)

2.1 The Differential Approach to Adjoint Systems

The original control system equations are represented by a set of simultaneous differential equations written in matrix form, called a *state vector differential equation* as follows:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \text{ with } x(t_0) \text{ specified,} \quad (2.1)$$

which includes: (a) the derivative of a column matrix, $\dot{x}(t) = dx/dt$, representing the rate of change of state of the system; (b) an $n \times n$ square matrix, $A(t)$, which with another column matrix consisting of n -state variables, $x(t)$ (called the *original state-vector*) represents the system to be controlled; and (c) an $n \times p$ matrix, $B(t)$, which with a $p \times 1$ matrix of inputs, $u(t)$, represents the control vector which sends the system into a new dynamical state configuration. Thus the vector (matrix) difference equation depicted in eq. (2.1) relates the rate of change of state of the system to the current state of the system and the current input signals. This differential equation is inhomogeneous and, therefore, represents a nonautonomous system because of the existence of $B(t)u(t)$ —a time-dependent control (forcing) term. We want to solve this equation to see if the specified control vector will send the system from a given initial state at $x(t_0)$ to an intended final state at $x(t_f)$ over an intended goalpath (defined by the intended manner of approaching the intended target).

The solution to this system, called the *steering function*, is given by the inhomogeneous integral equation

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^t \Phi(t, s)B(s)u(s)ds \quad (2.2)$$

where $\Phi(t, t_0)$ and $\Phi(t, s)$ are the state-transition (or fundamental) matrices of the free (autonomous) system given by

$$\dot{x}(t) = A(t)x(t) \quad (2.3)$$

defined over the interval $[t_0, t_f]$. Associated with the system depicted by eq. (2.1) is an observation vector $y(t)$, an m component vector which satisfies

$$y(t) = H(t)x(t) \quad (2.4)$$

where $H(t)$ is an $m \times n$ matrix relating the observation vector $y(t)$ to $x(t)$.

Before presenting the associated adjoint equations of information, one should note that the notion of adjointness is strongly dependent on the given space within which it is defined. Here the adjoint system is presented without any generic definition being given. (However, a specific definition is given in Appendix A.)

If the system of control equations is real, the adjoint system associated with eqs. (2.1, 2.4) is given by

$$\dot{\alpha}(t) = A^T(t)\alpha(t) + H^T(t)v(t) \quad (2.5)$$

and

$$z(t) = B^T(t)\alpha(t) \quad (2.6)$$

$\alpha(t)$ is specified and eqs. (2.5, 2.6) are integrated backwards in time. The superscript 'T' indicates matrix transpose (or its conjugate in the complex case). One can now define the dual properties of system eqs. (2.1, 2.4; 2.5, 2.6), such as, *complete controllability* and *complete observability*, by which the role of action and perception in a goal directed (intentional) behavior can be modeled. In addition, one can also define the *inner product operator*, the means by which perceptual information can be scaled to the control of action.

2.2 Controllability, Observability, and the Inner Product Operator

Definition: The action of the system, represented by eqs. (2.1, 2.4), is *completely controllable* if there exists some input $u(t)$ which takes the system from any initial state $x(t_0)$ to any other state $x(t_f)$ in a finite length of time $t_f > t_0$. This property holds if the following matrix is nonsingular for some $t_f > t_0$

$$W(t_0, t_f) = \int_{t_0}^{t_f} \Phi(t_f, t) B^T(t) B(t) \Phi^T(t_f, t) dt. \quad (2.7)$$

The measure of complete controllability is related to the minimum amount of control energy $u(t)$ necessary to transfer $x(t_0)$ to $x(t_f)$ in $t_f - t_0$ seconds.

Of interest to determining the optimality of the control is the degree to which the amount of work done approaches the minimum. For this one needs an equation defining minimum energy:

$$\text{Min } E = x^T(t_f) W^{-1}(t_0, t_f) x(t_f). \quad (2.8)$$

Small values of $W(t_0, t_f)$ imply little controllability, since large amounts of energy are required to transfer $x(t_0)$ to $x(t_f)$ and conversely.

Perceptual information guides action; hence a duality must exist between the energy required for control and the information that provides the measure of control. Such a measure is guaranteed by the duality of complete controllability to complete observability. This condition is defined next.

Definition: A system's state path is said to be *completely observable* if it is possible to determine the exact value of $x(t_0)$ given the values of $y(t)$ in a finite interval (t_0, t_f) where $t_0 < t_f$. The original system represented by eqs. (2.1, 2.4) is completely observable if the following matrix is positive definite for some $t_f > t_0$:

$$M(t_f, t_0) = \int_{t_0}^{t_f} \Phi^T(t_f, t_0) H^T(t) H(t) \Phi(t_f, t_0) dt. \quad (2.9)$$

It is important to note that there is a close relationship between these system properties.

A system is completely controllable if and only if its dual (adjoint) is completely observable. (See Lemma 1 in Shaw & Alley, 1985; Shaw, Kadar, Sim, & Repperger, 1992, p. 21.)

Analogous to the case of minimum energy, one can ask what happens to information when the system successfully achieves control of action with respect to some goal. Given the duality of complete observability with complete controllability, then whenever energy is minimized information must be maximized. Thus, the measure of complete observability is related to the maximum amount of perceptual information as follows:

$$\text{Max Info} = y^T(t_f) M^{-1}(t_f, t_0) y(t_f). \quad (2.10)$$

We have now arrived at the famous Kalman Duality Theorem:

The Kalman Duality Theorem: Complete observability is dual with complete controllability.

Corollary: Therefore if energy is minimized, then information must be maximized.

The last item of interest is the inner product of the original system with its dual, for it provides a global measure of the amount of control exercised as compared to the amount of information detected over the task interval.

Definition: Inner Product Operator is a bilinear function defined over any pair of elements $(x$ and $y)$ of a vector space

$$\langle x, y \rangle = x^T y. \quad (2.11)$$

Using the above definition, the inner product over the states of the original system and its adjoint happens to be a dynamical invariant. In other words $\langle x, \alpha \rangle = x^T \alpha = c$ (a constant).

These results may be further generalized. They can be extended to systems with hereditary influences, sometimes called *systems with retardation, or time lag*. (For further details consult Shaw & Alley, 1985; Shaw, Kadar, Sim, & Repperger, 1992).

2.3 The integral approach

It is well known that all differential equations can be formulated as integral equations. Using the operator notation, the inverse relationship between the differential equations and integral equations is made even more transparent. For this reason, and for its simplicity, the operator formulation is used. Let us consider the following second order differential equation as an exemplary case.

$$L[y] = p(x) y'' + q(x) y' + r(x) y = g(x) \quad (2.12)$$

where L denotes the second order differential operator

$$L = p(x) \frac{d^2}{dx^2} + q(x) \frac{d}{dx} + r(x), \quad \text{that is } L: y(x) \rightarrow g(x). \quad (2.13)$$

From the operator formulation naturally emerges the idea of using the inverse L^{-1} operator to find the solution of a particular differential equation. The inverse operator will be an *integral operator*

$$L^{-1}[g(x)] = \int_a^b G(x, t) g(t) dt. \quad (2.14)$$

Recall the eq. 2.1 (here rearranged) for the actor's control system

$$\dot{x}(t) - A(t)x(t) = B(t)u(t) \quad \text{with } x(t_0) \text{ specified.} \quad (2.15)$$

Using the differential operator notation this takes the form $L[x(t)] = B(t)u(t)$.

The solution given by integrating the above differential equation (eq. 2.2) was (shown rearranged)

$$x(t) - \Phi(t, t_0) x(t_0) = \int_{t_0}^t \Phi(t, s) B(s) u(s) ds. \quad (2.16)$$

From this specific example, we can see the role played by the Greens function by going to a generic form: The G kernel function is called the *Green's function of the operator L* . For the given control equation, the inverse operator takes the form,

$$L^{-1}[B(t)u(t)] = \int_{t_0}^t \Phi(t, s) B(s) u(s) ds. \quad (2.17)$$

Here the $\Phi(t, s)$ plays the role of the G kernel.⁷

Consequently, the right hand side of eq. 2.14, the integral part, represents the superposition of the intrinsic, quantized influences localized within the scope of system's law, as expressed in the integral form. That is why the Green's function is often called the *influence function* (Greenberg, 1971). Unfortunately, in practical application there are severe difficulties with this technique (see Appendix A).

2.4 Self-adjoint System Equations

Why is the adjoint system not adequate as a way of modelling the perceiving-acting cycle? Because adjoint system equations have terms representing sources of extrinsic influence. We need to make a transition from *adjoint systems* with extrinsic influences to a stronger form of adjoint systems, namely, to *self-adjoint systems*. To achieve the self-adjoint form, however, one must not only get rid of the extrinsic sources of influence but satisfy certain symmetry conditions as well. Self-organizing systems are *conditionally isolated*; that is, they sometimes act solely in accordance with intrinsic constraints because they are *self-adjoint*. (But take care, the physics of adjoint systems as compared to self-adjoint systems is complicated. Here we have used a simplified approach. For a full discussion of the issues, see Santilli, 1978; 1983).

Definition: A system is *self-adjoint* if it coincides with its adjoint.

⁷ The Greens function technique provides a method for 'absorbing' a forcing function. We can indeed find a G function for the given L and g , such that $L(G) = \delta(x'-x)$, where δ is the Dirac delta function, then the solution $y(x)$ of the equation $L(y) = g(x)$ will be $y(x) = L^{-1}[g(x)] = \int G(x', x) g(x') dx'$. To illustrate: given a differential equation, where $g(x)$ is a forcing term on the otherwise homogeneous, $L(y) = 0$ equation, $G(x', x) g(x') dx'$ now replaces the extrinsic forcing term.

How one might obtain the self-adjoint equations for a conditionally isolated system? They may be obtained using Greens function technique and selecting the proper transformations (Santilli, 1978; 1983). Shaw & Alley (1985) formulated the information and control relationships between an organism and its environment as a dual pair of dual integral equations (See Table I). These are self-adjoint integral equations because they have symmetric kernels. The 2×2 symmetry of these kernels represents the bi-directional propagation of information and control over the actor's perspective and the environment's perspective (see Table II). In psychological terms, these have been identified as *propriospecific* (organism referenced) and *exterospecific* (environment referenced) forms of information and control. And in addition to these, one can also identify their interaction terms (see Shaw, Kugler, & Kinsella-Shaw, 1991).⁸

(Insert: Table I: The Integral Equations Representing the Perceiving-acting cycle)

(Insert: Table II: The Adjoint Operators Representation of the Perceiving-acting cycle)

2.5 Why a Quantum Approach is Preferred over the Classical Approach

So far we have presented only one half of the "story", namely, we discussed dual adjoint systems rather than the dual pair of dual systems. The need for the four component subsystem equations suggests that the underlying structure is the *complex involution group*. This is one of several motivations that lead us from the classical adjoint-control theory to the complex Hilbert spaces and the quantum theory of psychological ecosystems thereby entailed.

Another motivation for moving to a quantum mechanical interpretation of intentional dynamics can be understood from Feynman's attempt to answer a problem with the classical approach raised by Poincare' (1905/1952). In Chapter VII of *Science and hypothesis* and echoed by many others ever since, Poincare' remarks in passing that the assumption of the *principle of least action* by which one passes from force-based mechanics to a potential (energy)-based mechanics involves an offense to the mind:

"The very enunciation of the principle of least action is objectionable. To move from one point to another, a material molecule, acted upon by no force, but compelled to move on a surface, will take as its path the geodesic line—i. e., the shortest path. This molecule seems to know the point to which we want to take it, to foresee the time it will take to reach it by such a path, and to know how to choose the most convenient path. The enunciation of the principle presents it to us, so to speak, as a living and free entity. It is clear that it would be better to replace it by a

⁸ These integral equations are directly related to the differential equation approach to adjoint systems by Shaw, Kadar, Sim, & Repperger, 1992).

less objectionable enunciation, one in which, as philosophers would say, final effects do not seem to be substituted for acting causes" (p. 128-129).

If one replaces the word 'molecule' with the word 'actor', then hardly a better description of a system with intentional dynamics is to be found anywhere. But here it seems inappropriate, for it amounts to anthropomorphizing inanimate particles. The danger that the variational approach to mechanics might tempt theorists to anthropomorphize particles is still recognized today. Feynman, who developed a version of quantum mechanics which addresses this issue expressed this problem:

"It isn't that a particle takes the path of least action but that it smells all the paths in the neighborhood and chooses the one that has the least action by a method analogous to the one by which light chose the shortest time" (Feynman, Leighton, & Sands, 1968; p. 9, chpt. 19).⁹

Our goal in the following sections is to show how a move to a quantum mechanical approach removes the 'offense to the mind' that concerned Poincare' and others. It does so by making plausible the thesis that behaviors of particles follow probabilistic waves rather than having a simple location on a force gradient. Through constructive and destructive wave interference the set of possible trajectories of the particle coalesces around the classical path of least action, indicating, not where the particle is, but where it is most likely to be found. The outcome of this move to quantum field theory is that the particle is constrained to the path observed, making it unnecessary for the particle to select its own path. Here, however, determinism (simple location and certainty) is traded off in favor of a tolerable degree of indeterminism (distributed location and uncertainty).

Formally, we need to provide the generic mathematical description of an organism with a complex interior, being driven by internally produced forces and guided by externally available information onto a goalpath toward a future goal-state. This image of a complex animate 'particle' exhibiting intentional dynamics in a field of information and control replaces the standard image of a particle with a simple interior, being driven by outside forces onto a 'least action' path that is indifferent to any future goal state.

The move to the quantum mechanical approach, vis a vis the Feynman path integral, provides a way to conceptualize how a particle 'selects' the classical stationary path (up to Planck's constant). We shall use this technique to explain how an actor having access to a field of information and

⁹ Also consider the more recent quote: "The mechanism by which the particle selects the physical trajectory of stationary action is not at all clear. The initial velocity is *not* given, so that the particle will not 'know' in which direction to start off and how fast to go. It is not clear how the particle can 'feel out' all trajectories and 'choose' the stationary one. It should be kept in mind that classical physics does not recognize any path other than the stationary path. Thus, out of a whole set of 'nonphysical' paths, introduced *a priori*, the classical principle of stationary action selects a unique physical trajectory through some mechanism which is not readily apparent" (Narlikar & Padmanabhan, 1986; p. 12)

control selects the goalpath it does. Only here, the global field constraints express, to a large extent, the intention of the actor rather than just the forces imposed on the 'particle' by the environment. The actor, through detection of information about an intended affordance goal, selects the boundary conditions (the Ω -cell) for the field by which it controls its action, by means of the effectivity engaged. The field is the bounded *Einstellung* ('determining tendency' plus boundary conditions) authored by the actor's intention, under the appropriate affordance-effectivity compatibility condition.

The inanimate particle, on the other hand, has to take whatever field that nature hands it. Put differently: The relevant wave which coalesces around the 'animate particle' is not merely a focus of global forces that completely controls its actions, but rather a knowledge wave, consisting of information as well as forces that allows it informed control. In more psychological terms, this explains how intentional dynamics can situate the perceiving-actor in an intentional context (an *Einstellung* = an Ω -cell).¹⁰

3.0 Quantum Mechanical Approach to Intentional Dynamics

The new strategy, which we propose to adopt, originates from a unique approach to quantum mechanics suggested by P.A.M. Dirac and developed by Richard Feynman (Feynman & Hibbs, 1965). This new approach, called *the Feynman path integral*, involves the formulation of the quantum mechanical behavior of particles in terms of *generalized, or distribution, functions* (Schwartz, 1950, 1951). Distribution functions (e.g., Dirac delta function or Heaviside function) are defined only under integrals. The Feynman distribution function¹¹ is defined under a special class of integrals that describe the sum over all possible path histories that a given particle *might have had!*

Our thesis is that perceiving-acting systems follow paths chosen from among a family of possible paths in the same manner that Feynman particles do. There is a major difference, however. For systems that are not just causal, as particles are, but are both causal and intentional, as perceiving actors are, then we must not only sum over their possible path histories but, dually, over their possible future paths as well. This is the way that controllability (i.e., causal) and observability (i.e., intentional) are represented under the Feynman path integral approach. By

¹⁰ Let's be clear about what claims we are making about the ontological status of the 'knowledge wave' field that is encompassed by the Ω -cell and set up by the actor's intention. It is not objective in the sense of being *in* the environment; nor is it subjective, in the sense of being a cognitive 'map' or other mental construct. Surely, this field is causally supported by both neurodynamical and physical processes, and structured by psychological processes—the 'determining tendencies' (e.g., values, needs, beliefs, etc.) of the actor. In this sense, it is functionally defined at an *ecological scale* which comprises all these processes.

¹¹ In the equation for the Feynman path integral, the distribution function, $Dx(t)$, replaces the ordinary $d(x)$. See eq. (3.33). Later, in dualizing the path integral for the purposes of intentional dynamics, $Dx(t)$ will be interpreted as $Dx(t)$ which is to be interpreted as being simultaneously $Dx(+t)$ and $Dx(-t)$, that is, as running in both temporal directions over all paths in the distribution.

dualizing this distribution function to express the temporal bi-directionality of information and control, the Ω -cell is automatically and necessarily obtained.

It is well-known that all the properties of quantum mechanics can be derived from either the Schrödinger wave equation approach, the Feynman path integral approach, or Heisenberg's matrix approach. The formal relationship of the differential approach and the integral approach to quantum mechanics is the same as the relationship of self-adjoint differential equations and the symmetric kernel integral equations.¹² (We shall be interested, for reasons that will become apparent later, in comparisons of only the first two approaches.) In other words, the differential (wave) equation and the (path) integral equation approaches provide formally equivalent descriptions. There is, however, an important difference between the two approaches. In the first case, the differential wave equation is a generalization of Newton's laws giving a *step-by-step* development of a particle's path in a manner that confounds dynamics and initial conditions. In the second case, the path integral approach is a generalization of Hamilton's variational approach giving a *path-by-path* account of a particle's possible histories, but in a manner that allows dynamics and initial conditions to be separated.¹³

Hence if the conditions that initialize (or finalize) a path are to be studied independently of the dynamic laws by which the path unfolds (causally by control or anticipatorily by information), then it is advisable to assume the path integral approach and to derive the wave equation applications from it. In this way, the probability amplitudes might still be of service, as we shall see, and one avoids loss of separation of boundary conditions from the dynamics. That is, it allows the boundary conditions on intentional dynamics, the Ω -cell, to be treated as a separate but related problem.

3.1 The Differential Approach to the Adjoint Quantum Mechanical Model

Thus, depending on the problem, we can use either of the two methods in formal analysis. However, to make clear the modelling lesson to be learned from quantum mechanics, and how quantum mechanics relates to the adjoint systems approach, the differential approach proves

¹² It should also be noted that there is a formally analogous relationship between the Hermitian (self-adjoint) matrix approach of Heisenberg's matrix mechanics and the adjoint differential equation approach of Schrödinger's wave mechanics.

¹³ Though mathematically equivalent, the wave equation and path integral equation are not physically equivalent. The function $\psi(x, t)$ depends both on the dynamics and on the initial condition $\psi(x_1, t_1)$. As shown in eq. (3.1), there is no way to separate these two aspects of a physical description. Narlikar & Padmanabhan (1986) explain it this way: "There exist physical situations in which we would like to study dynamics of the system without committing ourselves to any particular initial conditions. The kernel is the most suitable for such cases, since [eq. 3.4] clearly separates out the initial conditions from the dynamics. In short, the kernel is independent of the initial conditions and represents the dynamics while the wave function $\psi(q, t)$ depends on both the dynamics and the initial conditions" (p.25). (Equation numbers are for the current paper.) Also, where no wave equation can be constructed that is independent of end-points, a path integral can be. (See Feynman & Hibbs, 1965, for the construction of such a path integral.

simpler and more convenient. Most importantly, adjointness belongs to properties in the Schrödinger equations that are more transparent. To illustrate this point, examine the Schrödinger equation (3.1) for a particle moving in one dimension in a potential field and then compare eq. (3.2) With it's (complex) adjoint in eq.(3.3.):

$$-\frac{\hbar}{i} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \quad (3.1)$$

This equation is one specific form of the general Lagrangian

$$\frac{\hbar}{i} \frac{\partial \psi}{\partial t} = H\psi \text{ or equivalently } i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad (3.2)$$

where H represents an operator, called the *Hamiltonian* operator. Similarly, their complex conjugate (adjoint) equations

$$\frac{\hbar}{i} \frac{\partial \psi}{\partial t} = H^* \psi \text{ or equivalently } -i\hbar \frac{\partial \psi}{\partial t} = H^* \psi \quad (3.3)$$

can also be formulated. (Note: The '*' is used in quantum mechanics to denote complex conjugacy.) For the Schrödinger equation, the generic form of the associated integral equation will be (see more detailed discussion in Section 3.5)

$$\psi(x_2, t_2) = \int K(x_2, t_2; x_1, t_1) \psi(x_1, t_1) dx \quad (3.4)$$

where

$$K(x_2, t_2; x_1, t_1) = \int_1^2 e^{(i/\hbar)S(x_2, t_2; x_1, t_1)} D\mathcal{X}(t) \quad (3.5)$$

This equation provides the standard way to show the equivalence of the differential and the path integral approaches. By simple differentiation in (+) time yields the Schrödinger equation. (For ease of presentation, the derivation of the dual Schrödinger equation is suppressed; it should be clear, however, that it is obtained in parallel fashion from the dual version of eqs. 3.4 by differentiation with respect to -t.)

The differential operator of the (dual) Schrödinger equation is defined on the wave functions as their solutions. The nature of the equation implies that the Schrödinger differential operator maps solutions into other solutions, that is the domain and the range of the operator consists of wave functions only. This implies that, technically speaking, unlike what is usually the case, the

integral equations are not more difficult to handle than the differential equations. (The discussion of how the adjoint equation relates to observation/measurement will be discussed later).

Obviously, these quantum equations (eqs. 3.1, 3.2, 3.3) generalize the form of the classical core eq. (2.3) of the dual system of linear differential eqs. (2.1-2.6) that were used in the adjoint control approach. However, there are two major differences: the treatment of both the forcing functions and the boundary conditions will be much simpler in quantum mechanics.

Consider the role of forcing functions. In the classic case $x(t) = A(t)x(t)$ is called a *free system* meaning that it is *autonomous*. The generic form of control eq. (2.1), however, contains an extrinsic $B(t)u(t)$ term. By contrast, in the Schrödinger equation, there is no extrinsic term, that is, the extrinsic forcing components are formally not separated. Rather the forcing factors are automatically absorbed into the H operator.¹⁴ Nevertheless, this strategy is not without cost, for the Hamiltonian operator can take rather complicated forms.¹⁵ Regarding the boundary conditions similar arguments can be made.

3.2 Controllability, Observability, and the Inner Product in Quantum Mechanics

As in classical adjoint systems (Section 2.1), an inner product can also be defined on the space of the wave functions in the usual way. But here we must take a different route to interpreting observability and controllability. The fundamental problem here is that the Hamiltonian cannot be separated into an *informing* part and a *controlling* part.¹⁶ This is the price paid for the simplicity of adjointness in quantum mechanics, as compared to classical adjoint systems theory, where control and information could be separated. In other words, only a weaker form of the Kalman Duality theorem holds in quantum theory. Furthermore, it assumes an implicit rather than an explicit form. Thus, although the discussion of information and control in the quantum case must differ from the classical case, the key to the adjointness property in both cases is the *inner product* concept. For these reasons, we begin our discussion with the inner product operator.

¹⁴ In cases where the system can be conceived as an almost isolated one (with H_0 Hamiltonian), and there is a weak interacting component (with small H_{int}), then the Hamiltonian of this system can be written as a sum of the two parts $H = H_0 + H_{int}$. Even from this splitting, it has to be transparent that any external influence which changes the system, is modelled with a sudden absorption by using an additive interactive component.

¹⁵ As a simple example of this fact, consider a charged particle moving in a magnetic field.

$$-\frac{\hbar}{i} \frac{\partial \psi}{\partial t} = -\frac{1}{2m} \left(\hbar \nabla \psi - \frac{e}{c} \mathbf{A} \right) \cdot \left(\hbar \nabla \psi - \frac{e}{c} \mathbf{A} \right) \psi + e \phi \psi \quad (3.4)$$

where e is the charge, c is the velocity of light, $\mathbf{A}$ is a vector potential, and ϕ is a scalar potential. Even for this rather simple case, the Hamiltonian is quite complicated; namely,

$$H = -\frac{1}{2m} \left(\hbar \nabla \psi - \frac{e}{c} \mathbf{A} \right) \cdot \left(\hbar \nabla \psi - \frac{e}{c} \mathbf{A} \right) \psi + e \phi$$

¹⁶ Even if, under strong simplifying assumptions, one could separate the Hamiltonian for a single particle in a field, for a particle with a complex interior this will not be possible.

In the classical case the inner product is a *bilinear form* over two finite dimensional vector spaces that is formally a finite sum. By contrast, in the Hilbert space of quantum mechanics, defined over continuous functions, the inner product operator takes an integral form.

Definition: Let f and g be two probability amplitude (wave) functions, then $\int g(x) * f(x) dx$ is called the *inner product* of f and g .

The inner product operator is closely tied to a given quantum mechanical system, that is, to its Hamiltonian. How can we unpack this inner product operator to reveal observability and controllability as separate factors? Unpacking the inner product operator will have profound implications for how one interprets the perceiving-acting cycle as situated in an intentional context. The intentional context will be modelled by the Hamiltonian of the system, while the perceiving (observability aspect) and the acting (controllability aspect) will be represented by operators with special properties being required. The tight relationship between perceiving and acting will be revealed as operators that are self-adjoint, that is, the same.

Assume that perception involves a *meter* and that action involves an *effector*, then this self-adjointness property implies that such mechanisms are but different aspects of the same operator. Though the idea is not fully developed here, self-adjointness suggests a possible formal characterization of the construct of a 'smart perceptual device'—a kind of ecological (inner product) operator (Runeson, 1977). Here an actor's capacity for acting (an effectivity) and metering are unified under the intention to discover some characteristic property (an affordance) of the local environment, as in wielding a visually occluded implement to determine its length and its suitability for use in some task (Solomon & Turvey, 1988; Turvey, 1989).

In quantum mechanics one does not have to worry about the specific conditions under which the self-adjointness (hermiticity property) requirements are satisfied. (Primarily, one is concerned with identifying the Hamiltonian of a system). In quantum mechanics the Hamiltonian is always Hermitian.

Definition: An operator H is called *Hermitian*, or *complex adjoint* if

$$\int (Hg) * f dx = \int g * (Hf) dx \quad (3.6)$$

holds with the property that any f and g converge to zero at infinity.

Replacing f and g with ψ in eq. (3.6), that is substituting the solution of the Schrödinger wave equation into f and g , we get

$$\int (H\psi) * \psi dx = \int \psi * (H\psi) dx \quad (3.7)$$

If ψ satisfies eq. (3.2)¹⁷ then from eq. (3.7) it follows by a substitution

$$\int \frac{\partial \psi^*}{\partial t} \psi dx + \int \psi^* \frac{\partial \psi}{\partial t} dx = \frac{d}{dt} \int \psi^* \psi dx = 0 \quad (3.8)$$

This important result shows that the inner product is time independent for a solution of eq. (3.2).

In the control theoretical framework, the perception-action system was formally modelled by dual differential equations expressing the observability-controllability conditions. In quantum mechanics, the control of action is expressed in terms of the Schrödinger equation of motion. Its dual process, the detection of information, is identified in quantum mechanics as measurement. This is not unusual, for the perceptual system has been treated as a measurement device before (e.g., Rosen, 1978; Bingham, 1988; Shaw, 1985). What kind of measuring process is perceiving and how does it relate to controlling? We discuss these issues next.

An analogy can be constructed between the influence of a scientist's measurement on the motion of an inanimate particle along its trajectory (in the laboratory frame of reference) and the influence of information detection on the perceptual control of an actor's (self-)motion along its goalpath (in the Ω -cell frame of reference). This analogy holds but with qualification so that measurement of the particle becomes perception by the 'particle' and extrinsic control becomes informed self-control. Therefore, with certain requisite modifications, the mathematics of quantum measurement can be extended to the case of a complex particle exercising self-control from the case of a simple particle subject to extrinsic control.

In classical quantum mechanics, the measurement process is limited to a short period of time. But for perception (and control) within an intentional context (Ω -cell) the process is continuous between boundary conditions (intent to target at either the α , ω , or Ω scale, as shown in Figure 3). Measurement (and therefore perceptual) information can be represented in the Schrödinger equation of control. The Hamiltonian for the simple particle can be generalized to include a component representing the influence of information on the control of motion (Shaw, Kugler, & Kinsella-Shaw, 1991). How might this be done? Information can be conceived as a field, and the goal for a given task can be modelled as an attractor in the information field (Kugler, Shaw, Vicente, & Kinsella-Shaw, 1990). (But see footnote 4 for qualification). As our simple example, consider again a charged particle moving in a magnetic field. (See footnote 14.) In the formulation for this problem, an external field can influence the form of the control equation without changing the generic form of the Schrödinger equation. Because of their duality, this suggests that the

¹⁷ In quantum mechanics, unlike the classical adjoint systems approach, time-reversal cannot be modelled simply by the adjoint system, rather, as proven by Wigner (1932), the correct time-reversal transformation sets $T\psi(t)$ equal to its complex conjugate so that $T\psi(t) = \psi^*(-t)$ rather than setting it to the simpler adjoint, $T\psi(t) = \psi(-t)$.

information field can be modelled within the quantum mechanical framework in a similar fashion. Using the (dual) Schrödinger wave function to characterize the perceiving-acting cycle as a 'knowledge wave', contrasts sharply with the traditional view of it as negative feedback control (Smith & Smith, 1987).

To appreciate the modelling strategy for introducing observability (and hence controllability) into quantum mechanics, consider the nature of measurement in quantum mechanics more closely. Assume a measuring device M measures a property G of a moving particle. Property G is called an *observable*. More specifically:

Definition: An *observable* is a Hermitian quantum mechanical operator G .

Definition: The *expectation value*, $\langle G \rangle$, of G in the state f is defined by the integral

$$\langle G \rangle = \int f(x)^* G f(x) dx = \int (G f(x))^* f(x) dx. \quad (3.9)$$

Definition: A *measurement* is the expectation value of an observable G .¹⁸

Practically speaking, the measuring device must complete the measurement in finite time. One of the most interesting aspects is that measurement changes the wave function of the 'otherwise freely moving particle' to be measured. Formally, one can show that if the wave function of the incoming particle is $f(x)$, then the measuring equipment modifies the kernel K of the amplitude

$$f(x_2, t_2) = \int K(x_2, t_2; x_1, t_1) f(x_1, t_1) dx \quad (3.10)$$

by making it equal to $K_{exp}(x_2, t_2, x_1, t_1)$ in the course of measurement starting at $t = t_1, x = x_1$, and ends at $t = t_2, x = x_2$. The inner product of f and K_{exp} that is $\int K_{exp}(x_2, t_2, x_1, t_1) f(x_1, t_1) dx$, gives the amplitude to arrive at x_2 at the outset of the equipment.

$$fG(x_2, t_2) = \int K_{exp}(x_2, t_2; x_1, t_1) f(x_1, t_1) dx \quad (3.11)$$

Using Feynman's notation (Feynman & Hibbs, 1965), the probability function associated with the property G will be

$$P_G(x_2, t_2) = \left| \int K_{exp}(x_2, t_2, x_1, t_1) f(x_1, t_1) dx \right|^2 \quad (3.12)$$

where $f(x)$ is the wave function to be measured, $K_{exp}(x_2, x)$ is the kernel for the experimental apparatus, and x_2 is the position arrived at by particles with property G . This result, however, depends on experiment (including the measuring equipment and experimental conditions, such as

¹⁸ Since G can be measured it must be real, that is G must be a Hermitian (complex conjugate) operator.

duration of measurement, etc.). To find the kernel of another experiment, another form, $K_{exp'}(x_3, x)$ is needed:

$$P_G(x_3, t_3) = \int K_{exp'}(x_3, t_3, x_1, t_1) f(x_1, t_1) dx_1^2 \quad (3.13)$$

Since the same property G is measured in each experiment, $P(G)$ should be the same for any incoming wave function $f(x)$ within an unimportant constant phase factor $e^{i\delta}$. It follows that

$$K_{exp}(x_2, t_2, x_1, t_1) = K_{exp'}(x_3, t_3, x_1, t_1) = g(x) \quad (3.14)$$

so that one obtains an experiment-independent form of the kernel. This independent function, $g(x)$, is called the *characteristic function* of the property G . One can now refer to the quantum mechanical analogue of observability used in classical adjoint control system. Notice that the integral in eq. (3.13), generally, yields a complex number for the measured amplitude. Furthermore, if the measurement is expressed as a G transformation on the incoming wave function f , then eq. (3.13) can be written in the simplified form

$$P(G) = \int f(x)^* G f(x) dx^2. \quad (3.15)$$

The integral will be real if the G operator of the observable is Hermitian. (Compare this result with the required positive definiteness for M in eq. 2.9). This provides the basis for requiring G to be Hermitian operator

Knowing the limitation of measurement in quantum mechanics due to the uncertainty principle, one should not expect a definition of *complete observability*. Nevertheless, a more general definition, called *maximal observability*, can be formulated. This suggests that there may only be an approximate generalization of the Kalman Duality theorem to quantum mechanics. We are not yet clear whether there is a *mini-max* duality between eqs. (3.16) and (3.16a) as there is in the classical adjoint systems case (Section 2.2). This possibility should be ascertained.

Definition: G property is *maximally observable* in a quantum mechanical measuring system if G is Hermitian and

$$P(G) = \int f(x)^* G f(x) dx \rightarrow \max \quad (3.16)$$

Here G must be a close approximation to the generalized Hamiltonian of the system defined over the Ω -cell. Having the intrinsic adjointness of the quantum mechanical equations and the equations of a measuring system, the corresponding definitions for controllability may also be formulated. This can be done by the appropriate variational principle. In quantum mechanics the variational principle is called the *Rayleigh-Ritz method*. It states that if H is the Hamiltonian of the system with E_0 as the lowest energy state value, then for any f the following condition holds:

$$E_0 \leq \int f(x)^* H f(x) dx / \int f(x)^* f(x) dx \quad (3.17)$$

This form is not really helpful for our purpose. The fundamental problem faced is that here one wants to split the Hamiltonian into an *informing* and a *controlling part*.¹⁹ In other words, here one must pay the price for the simplicity of the quantum mechanics as compared to the classical approach, where the control and information parts were given in separate equations. Imagine, for example that we need to provide a field to control the path of a particle. Then the controllability can be defined on the basis of the Hamiltonian, which includes the 'control field'. Unfortunately, the measurement (observation) will change the Hamiltonian of the system. Consequently, it is not possible to isolate the control part of the new Hamiltonian. There is a kind of tautological limit on what one can do to separate control from information in the quantum case. One can take a control perspective or an information perspective on the actor's generalized Hamiltonian but there is but one quantity. Hence these control and information seem quite tautological. To get around the tautological nature of the control versus the measurement problem, it seems to us that one can do no better than to consult Feynman & Hibbs' (1965) discussion of the issue. In their discussion, this tautological nature of information and control is simply a strange property of the characteristic function.

They initiate the discussion with the following question: What is the relationship between f and g ? Before answering this question, one must ask: What should the *state function* f be to have the property G ? To find a particular state function, F , for a given experimental apparatus i with a given characteristic function g , one has to solve

$$\int K_{exp}(x_i, x) F(x) dx = \delta(x_i - x). \quad (3.18)$$

This equation has the well-known solution $K_{exp}^*(x_i, x)$ for $F(x)$. Here $K_{exp}^*(x_i, x)$ is the complex conjugate of $K_{exp}(x_i, x)$. Consequently,

$$F(x) = K_{exp}^*(x_i, x) = g(x). \quad (3.19)$$

That is, $g(x)$ gives us the wave function of a particle having the property G with probability 1. Furthermore, if the particle is in state $f(x)$ the amplitude that it can be found in a state $g(x)$ is

$$\psi(G) = \int g(x)^* f(x) dx = \Phi[g(x)] \quad (3.20)$$

¹⁹ Here one might expect the duality property (under a Greens function) of the time-forward Feynman propagator and the time-backward Dyson propagator, might be useful ways to represent controlling and informing, respectively. Unfortunately, the problem is more complex than this, for one must have coupling of information and control over internal and external frames of reference. Recall the quote in Section 1 from Shaw & Kinsella-Shaw (1988). These issues, however, have been touched upon algebraically (but not explored analytically) in Shaw, Kugler, & Kinsella-Shaw (1991).

Having outlined the major point of Feynman & Hibbs' discussion (1965, pp. 96-108), they conclude:

"We might say loosely: The probability that the particle is in the state $g(x)$ is $|g(x) f(x) dx|^2$. This is all right if we know what we mean. The system is in the state $f(x)$, so it is not in $g(x)$; but if a measurement is made to ask if it is also in $g(x)$, the answer will be affirmative with probability

$$P(G) = \int |g^*(x) f(x) dx|^2 = \rho[g(x)] \quad (3.21)$$

A measurement which asks: Is the state $g(x)$? will always have the answer yes if the function actually is $g(x)$. For all other wave functions, repetition of experiment will result in yes some fraction P (between 0 and 1) of the tries. This is a central result for the probabilistic interpretation of the theory of quantum mechanics.

For all this we deduce an interesting inverse relationship between a wave function and its complex conjugate. In accordance with the interpretation . . . [see eq. 3.20], $g^*(x)$ is the amplitude that if a system is in position x , then it has the property G . [Such a statement is put mathematically by substituting a δ function for $f(x)$ [see eq. 3.20]. On the other hand, $g(x)$ is the amplitude that if the system has the property G , it is in position x . (This is just a way of giving the definition of a wave function.) One function gives the amplitude for: If B , then A . The other function gives the amplitude for: If A , then B . The inversion is accomplished simply by taking the complex conjugate. Equation [3.21] can be interpreted as follows: The amplitude that a system has the property G is (1) the amplitude $f(x)$ that it is at x times (2) the amplitude $g^*(x)$ that if it is at x , it has property G , with this product summed over the alternatives x ." (Feynman & Hibbs, 1965; pp. 108-109. Numbering on equations refer to equations in the current paper.)

The gist of this section can be interpreted as the quantum version of what Gibson called a *rule for the perceptual control of action* (1979). For the measuring process (detection of goal-specific information) to be successful (in controlling action), the g characteristic function (affordance goal property) of the environment has to be a complex conjugate of the state function f (the effectivity property) of the actor. The measurement procedure specifies a characteristic function (goal-specific information) which will be a real extrinsic constraint. For the measuring process to be successful, the free particle should modify its state function as a result of measurement. More specifically, in order to have a good measurement (high probability) the self-adjointness (or complex conjugacy) of the characteristic function and the state function of the moving particle should be properly set up. The calibration includes the boundary conditions (the Ω -cell) within which the device executes its measurement.

In the quantum mechanical example of a moving charged particle in the magnetic field, the focus was on the measurement problem. Actually, both the action of the particle and the measuring process are equally important, even though the role of action is essentially implicit in the discussion. (This is in keeping with Gibson's being a perceptual psychologist who was at heart an action psychologist.) This is the natural consequence of the fact that the particle is not an intentional system which can set its own goal parameters. In the case of a living organism the focus should be on the goal-directed action which is guided by perception. That is, the focus should be on the intentionally selected goal specific action guided by perception.

For an organism moving toward an intended goal, and perceiving (measuring) its state relative to the goal (which is given by the characteristic function), the task is to move so as to maintain its self-adjointness. This self-adjointness is achieved by the actor observing a rule for the perceptual control of action; namely, in the language of quantum theory: 'Move so that the wave function of motion is the complex conjugate of the characteristic function given by the perceptual measurement!'; and, in the language of intentional dynamics: 'Move so as to perceive what you need to perceive if you want to satisfy your intention of maintaining your goal (that is, completing your intentional task)!' (See Section 3.5 for further discussion).

In the above discussion, intentional dynamics assumes that the property G is an approximation of the generalized Hamiltonian that must be defined over the whole Ω -cell. This compact theoretical formulation may be both too brief and too ambitious. For it requires perfect knowledge of all the observables with regard to the given Hamiltonian—something usually not known explicitly. Nevertheless, it seems clear that a quantum theoretical framework for intentional dynamics may be in the offing. Final judgment should be suspended until empirical examples have been thoroughly worked out. (Note: There are other observables defined with respect to the Ω -cell of intentional dynamics, and additional conditions from quantum mechanics to be satisfied. These are discussed in the Appendix B).

To complete the parallel presentation of the quantum approach to the classical case (Section 2), the integral equations of quantum mechanics formulated by Feynman must be introduced. We do so in the next section.

3.4 Feynman Path Integral Approach

The equivalence of the Feynman path integral formulation and the conventional presentation of quantum mechanics by the Schrödinger differential equations is discussed in several books. The translation between the two languages can easily be found in the literature. For instance, the detailed analysis of the transition from the path integral to the Schrödinger differential equation can be found in Feynman's book (Feynman & Hibbs, 1965). The most common way to do the translation is to differentiate the equation of the path integral so as to derive the Schrödinger

equations. Obviously, the derivation is invertible. The inverse direction of translation can also be done by simply reversing the derivation steps. However, here we present the translation from the differential approach to the path integral approach for two reasons:

First, because certain important aspects of the path integral formulation help us understand more about the usefulness of the quantum mechanical technique in intentional dynamics. Second, because it is important to show the role of Green's function and its generalization for the quantum mechanics. Partly, because the self-adjoint formulation in control theoretical framework (Shaw, Kadar, Sim, & Reppeger, 1992) naturally offered the Green's function as a candidate to understand formally, and perhaps also empirically/physically the underlying deeper processes.

Following the same steps of the above construction of integral equations as inverse ones to the differential equations, the derivation begins with writing the Schrödinger equation. Next, one must find the corresponding Green's function that will be the kernel of the integral equation associated with the Schrödinger equation. Finally, it must be shown how one can obtain the Feynman path integral formulation from the Greens function.

For the Schrödinger equation, the generic form of the associated integral equation will be

$$\psi(x_2, t_2) = \int K(x_2, t_2; x_1, t_1) \psi(x_1, t_1) dx \quad (3.22)$$

This equation provides the standard way to show the equivalence of the differential and the path integral approaches. By simple differentiation in time, it yields the Schrödinger equation. (Again, to keep the presentation simple, we suppress the dual version of these equations).²⁰

We need to point out that the differential operator of the Schrödinger equation is defined on the wave functions as their solutions. As mentioned earlier, the Schrödinger differential domain of the operator the domain and range of the operator consists only of wave functions. Consequently, the inverse operator can be written in the form eq. (3.22) with the boundary condition

$$K(x_2, t_2; x_1, t_1) = 0 \text{ for } t_2 < t_1. \quad (3.23)$$

The Green's function of the Schrödinger equation will be the kernel of the integral eq. (3.22) as it can easily be seen by comparing eq. (2.21) with eq. (3.22). The Green's function represents a local (infinitesimal) influence resulting in displacement. The kernel $K(x_2, t_2; x_1, t_1)$ of the Schrödinger equation can be given explicitly as

²⁰ The original, time forward equation is given by eq. 3.31. The dual equation then is $\psi(x_1, t_1) = \int K(x_1, t_1; x_2, t_2) \psi(x_2, t_2) dx$. The dual kernel to eq. 3.33 (antipropagator) is

$$K(x_2, t_2; x_1, t_1) = \int e^{(i/\hbar)S(x_2, t_2; x_1, t_1)} D x(t) \text{ As a short hand for both the original and the dual equations,}$$

one might replace the usual path distribution functional, $Dx(t)$ with $Dx[t]$ meaning that the distribution is temporally bi-directional, i. e., is both $Dx(+t)$ or $Dx(-t)$.

$$K(x_2, t_2; x_1, t_1) = \int e^{(i/\hbar)S(x_2, t_2; x_1, t_1)} D x(t) \quad (3.24)$$

Eq. (3.24) is Feynman's path integral representing integration over all the possible paths between (x_2, t_2) and (x_1, t_1) . (Again, consult footnote 19). The kernel is also called *the amplitude with respect to its endpoints*. Figure 5 illustrates the way in which a classical (stationary action) path can be obtained from the distribution, $Dx[t]$, by constructive and destructive wave interference. Traditionally, the divisor in the exponential term, $\omega = \hbar$. This shows that the width of the uncertainty region around the classical path has the width of Planck's constant, $\hbar$. For generality, however, (explained below), this constant is replaced with a variable ω . Since this is key to understanding the origins of the 'knowledge wave', let's consider this process in more detail.

(Insert Figure 5: Emergence of the 'Knowledge Wave' within the Ω -cell)

We ask: How does a particle (or an actor) get from an initial point (intent) to a final (target) point? In the classical approach, although the principle of least action picks out the path, it is not clear how the particle is constrained to that path. Also, in conventional formulations of quantum mechanics, no definite path is possible because of uncertainty. Hence the path concept is deemed useless. Feynman's insight was to appreciate the positive import of this problem; namely, if a unique path is not possible, then *all possible paths are allowed!* Furthermore, he showed how the classical path could be recaptured: Weight each path by the factor $e^{iS/\hbar}$ including the classical path. Feynman then showed that each path is more or less in dynamical phase with the other possible paths. Thus they each contribute to the sum of amplitudes which is greatest in the vicinity where the classical path is to be found by variational techniques. More particularly, the classical path is distinguished by making the action, S , stationary under small changes of path: thus close to this path the amplitudes tend to add up constructively, while far from it their phase factors tend to cancel because of destructive wave interference. The path integral approach essentially gives a 'global' formulation to classical field theory, and for our purposes, to intentional dynamics.

To enrich the intuition on the meaning of the path integral, consider how it may be extended to a concatenation of path distributions (e.g., a sequence of ω -cells):

Amplitudes for events occurring in succession can be expressed in the form

$$K(x_2, t_2; x_1, t_1) = \int_c K(x_2, t_2; x_c, t_c) K(x_c, t_c; x_1, t_1) dx_c \quad (3.25)$$

where the integration means summing over all x_c, t_c points, that is the total amplitude to go from (x_1, t_1) to (x_2, t_2) is the sum of the product $K(x_2, t_2; x_c, t_c) K(x_c, t_c; x_1, t_1)$ taken for all

possible (x_c, t_c). The concatenated Ω -cell partitions, shown Figures 2 and 3, provide cases where the 'chain' rule of kernel products applies. It is important to recognize that the Feynman path integral is defined from initial (or final) point to a moving current point, which acts as a parameter that distributes action (S) over the paths moment to moment. Thus, the partitions depicted in Figures 2 and 3 arise dynamically as a function of the perceiving-acting cycle branching at different choice-points while leaving the overall intention (Ω -cell) invariant (For example, consider a predator who must change direction in order to continue tracking a dodging prey.).

(Insert: Figure 6: Showing the Range of Scales for the Weighting Function in the Feynman Path Integral.)

In the weighting function, $\exp(iS/\omega)$ (eq. 3.24), over the Feynman distribution, $Dx(t)$, or $Dx(-t)$, the scale used in physics is $\omega = h$ (Planck's constant). This weighting function can be generalized to ecophysics and applied to intentional dynamics. By replacing h with a variable scaling factor corresponding to α or ω , where $h \leq \alpha \leq \omega \leq \Omega$, one can have graded partitions of uncertainty (tolerance) around the classical stationary path. The existence of a variable scaling factor expresses mathematically Kugler & Turvey's (1987) claim that the action system can be variably quantized. Also, the total action (S) associated with these path partitions can be expressed as a product of kernels of the Feynman path integral as defined by eq. (3.25).

3.5 The Analogy Between Ecological Laws and Quantum Mechanical Laws

Classical mechanical laws apply to predict events: Given the appropriate initial conditions (i.e., the mass and layout of three balls A, B, and C) so that if event₁ occurs (e.g., ball A strikes ball B), then event₂ (i.e., ball B strikes ball C) necessarily (lawfully) follows. Traditionally, psychological laws have been assumed to take the same causal form: Given the appropriate initial conditions (i.e., normal organism with proper learning history, attending to stimulus, and so forth), then if event₁ occurs (a stimulus event), then event₂ (a certain response) probably (lawfully) follows. Here, as Skinner (1977) suggests, the stimulus, although not truly a force, acts like a force, and the control 'law' (next state function), although not truly a law, acts like a law to move the organism into its next state from which it emits the observed behavior. If the state transition is associative, then this form of law fits a *stimulus-response* behaviorism; however, if the state transition involves a *representation*, or *symbol*, then this form of law fits cognitive psychology (Fodor & Pylyshyn, 1988). This classical law form, however, fits neither quantum phenomena nor ecological psychology phenomena (e.g., intentional dynamics); rather, they both take a different law form.

It is generally agreed that quantum mechanical laws do not predict events with absolute certainty, as deterministic classical laws are supposed to; rather they predict only the probability that subsequent observations (measurements) will follow from previous observations

(measurements) if, as discussed earlier, a certain self-adjoint (complex conjugate) relationship holds between a state function and characteristic properties of the situation (Wigner, 1970). As indicated, ecological psychology requires laws that operate similarly.

Consider a rule for the perceptual control of action (Gibson, 1979), say, as formulated from the perspective of a prey engaged in a prey-predator competition. 'If you (the prey) intend to escape the predator, whose image is expanding in your optic array, then intend to move so as to make the predator's image contract!' Here, analogous to the quantum law formulation, the law relates a previous observation (information) to a subsequent observation. The quantum mechanical interpretation of intentional dynamics gives the following generic reformulation of a rule for action: 'If you (the actor) intend following one of the acceptable goalpaths (i.e., in a congenial Ω -cell distribution) having intended characteristic property g' (positive affordance value), then stop applying the old state function, f (an inappropriate effectivity), which generates unacceptable paths (i.e., in a uncongenial Ω -cell distribution) having the unintended characteristic property, g (negative affordance value), and begin applying a new state function, f' (an appropriate effectivity)!'

(insert Table III: A Comparison of Law Forms)

Table III compares the different laws discussed. Both forms of the classical law form (I and II) relate event to event, while the quantum-type law form (III and IV) relate information to information through a function that is the complex conjugate of the characteristic property of that information. In the quantum case, a state function does so, while in the intentional dynamics case, a path function (an effectivity) does so.

4.0 Conclusion

In the adjoint information/control theory (Shaw, Kadar, Sim, & Repperger, 1992), perception was formally construed as *observability* and the action as *controllability* (Kalman, Englar, & Bucy, 1962). This traditional law approach treats control systems as an analytic extension of classical mechanics, formulated in terms of ordinary differential equations, or, alternatively, as an extension of variational mechanics, formulated in terms of functional (Volterra) integrals. It was argued that although these mathematics are quite appropriate, up to a point, they have certain inherent limitations for modelling perceiving-acting systems which exhibit intentional dynamics (e.g., prospective control). The self-adjointness property is a merit of this traditional approach, but alone it is not sufficient. Rather the classical variational approach to defining the goalpath of the perceiving-acting cycle has inherent shortcomings because of the mathematical physics it inherits from classical mechanics. These three shortcomings are most prominent:

- (1) it does not give an account for how a 'particle' selects a stationary path;

(2) it does not provide a principled way to handle the tolerance limits on detection and control in following a path; and

(3) it provides no way to embed the perceiving-acting cycle in an intentional context.

Self-adjointness is a natural property of Hilbert space and so is inherited by quantum mechanics. Hence, it was argued that the perceiving and acting in intentional contexts might have a natural description in quantum mechanics so that these three shortcomings are overcome.

To the above complaints we gave the following remedies:

- (1)* The Feynman path integral provides a physical motivation for the claim that the path that a 'particle' elects necessarily emerges very close to the classical (stationary action) path. Figure 5 shows that the quantum action distribution is constant to the first order in the vicinity of the classical path (dark strip given by constructive wave interference), while outside this region the dynamical phase oscillates so erratically that the corresponding amplitudes of the other possible paths are washed out (by destructive wave interference).²¹
- (2)* The tolerance limits around the path represents the fallibility of control of information detection by intentional 'particles' and Planck's constant range of Heisenberg uncertainty around the path of inanimate particles (Figure 5). The tolerance range for 'particles' exhibiting intentional dynamics is variable, depending on the nature of the task, the degree of certainty of the intention held, scale of information and control resolution, and the number of interpolated choice-points. Regardless of these details, however, the weighting function in the kernel, $e^{iS/\hbar}$ (where $\hbar$ ranges from h to Ω) which propagates the path, unites intentional dynamics with quantum physics *vis a vis* the Feynman path integral and provides access to the Schrödinger wave function—the 'knowledge wave' in the case of particles exhibiting intentional dynamics. An important goal of ecological physics has been to provide a continuous link between psychology and physics (Shaw & Kinsella-Shaw, 1988). This link is now forged by this variable weighting factor, $\hbar$, for it shows how psychology, through intentional dynamics, can be continuous with physics when a constant scale factor is allowed to become a variable one.
- (3)* The perceiving-acting cycle becomes situated in an intentional context when it is embedded in an Ω -cell. Here not only is generalized action conserved under successful goal-directed behaviors but intention acts as a kind of implicit 'steering function' (prospective control) so

²¹ The Feynman integral is difficult to handle numerically because the trajectories, far from the classical trajectory having complex exponentials, oscillate rapidly (i.e., they have a negative definite metric). To allow for numerical approximations, a bridge from quantum mechanics to statistical mechanics can be built by rotating time t into an imaginary direction by the operation (a Wick rotation) $t \rightarrow i\tau$. This has the effect of dampening the wildly oscillating exponential, $e^{iS/\hbar}$, and turning it into an exponentially decreasing function, $e^{-S/\hbar}$, which behaves more like classical weighting functions (i.e., with a positive definite metric). Multiplying through by $-i$ acts holomorphically to counter-rotate the solutions to this path integral back onto the original metric (Aitchison & Hey, 1989).

that the actor (who is well-intentioned) can ride the crest of a dual (information and control) 'knowledge wave'. The symmetric kernel of the dual Feynman path integral, as defined over $Dx[t]$, is called a *propagator* because it generates all the knowledge (remembered history and anticipated future) that may exist for a 'particle' (actor). Consequently, it is the core equation for intentional dynamics as it is for quantum physics and provides the attractor dynamics needed to explain goal-directed behavior.

Perhaps, the chief failure of classical physics, and inherited by the classical adjoint control theory is to make room for goalpaths that are possible but not pursued. In the classical approach, the only path which exists is the path actually followed by a particle, in accordance with the principle of least action. For intentional systems, where choice behavior must be real, paths intended but not followed must be as real as paths that nature defines by least action. This requirement of intentional dynamics, like that of quantum mechanics, calls for a dramatic change in scientific philosophy.

Like quantum mechanics, intentional dynamics, needs a physics of *possibilism* and not just of actualism (Turvey, 1992). Fortunately, where classical physics does not allow for such possibilism to underwrite choice behavior, the newer view of quantum physics based on the Feynman path integral does. The implications for the philosophy of psychology are immense. Under the classical approach to perceiving and acting in intentional contexts, one had to posit nonobservable constructs, such as unconscious inference, magically acquired memories or cognitive maps, and so forth, to underwrite the choice set from which actual choice-behaviors emerge.

By contrast, from an ecologized version of the Feynman path integral, a field of information and control, is shown to emerge. By differentiating this integral, a Schrödinger-like 'knowledge wave' arises to illuminate the actor's goalpath choices—a goalpath distribution whose width is automatically scaled to the abilities of the actor by an intrinsic weighting function. If so, then the intentional control of the focus of attention, by which the actor finds its way, is explained.

In summary, our aim in this paper was to show that measurement in quantum mechanics could be extended to the Ω -cell to model the perceiving-acting cycle in an intentional context and to show that the existence of the so-called 'knowledge wave' is by no means fanciful. We have provided arguments in favor of both of these claims. How does this approach relate to other attempts to use quantum mechanics in psychology?

If, as we suspect, the internal mechanisms for perception, action, and cognition (e.g., intentional focusing of attention) under the auspices of this approach are to be replaced by ecological operators on a quantum-like field theory, one must wonder if any help from neurodynamics for such mechanisms is in the offing. Apparently so.

It is worth noting in this regard that variations on quantum mechanics have been used to model figural processing (Pribram, 1991), as well as audition (Gabor, 1946), at two distinct constituent scales in the analysis of brain processes: the macroscale of information processing in the brain and at the nanobiological scale of the microtubular processes of the cortex (Hammeroff, 1987). At the more macro scale, the neurodynamics that support visual and auditory perception have been framed in terms borrowed from quantum microphysics. More specifically, the activity at the level of the dendritic microprocesses has been modelled as a quantum field, where Pribram has hypothesized quantum or patch holographical processes to occur and 'signals' are "... better conceived of as Gabor-like elementary functions—*quanta rather than bits of information*" (Pribram, 1991, p.271). Where the above approach represents an extrapolation from quantum microphysics to neurodynamics, our efforts represent an attempt to develop a quantum macrophysics appropriate to intentional dynamics at the ecological scale. Where brains provide the boundary conditions for the former approach, the Ω -cell does for the latter (see Shaw, Kinsella-Shaw, & Kadar, *in preparation*).

We have surveyed the promise of the quantum mechanical approach to modelling the perceiving-acting cycle in an intentional context and found many ways that these mathematics might be appropriate. We have also discovered problems that must be overcome if the complex nature of the interaction of information and control is to be understood. Obviously, much further work is required.

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Appendix A: Constructing Green's Functions

There are several difficulties in constructing a Greens function for a given differential equation. First of all, the inverse does not necessarily exist. This shortcoming can be redeemed in many cases by some transformation. Even if the inverse exists, its construction is usually more difficult than finding solutions by using a properly chosen conventional trick. However, if the inverse exists, then the problem is equivalent to the task of finding or constructing the kernel, the Green's function of the integral equation (See e.g. $G(x, t)$ in eq. [2.14]). There is more than one way to construct the Green's function associated with a differential equation. Each method may have corresponding physical meaning. Here we presented the one which is the easiest and the most revealing in terms of using the adjointness we have already introduced in our paper. Here we just further refine the basic concepts.

Definition: L^* differential operator is *formally adjoint* to L if L and L^* are associated with the following equation

$$\int f^*(x) Lg(x) dx = [...] + \int f(x) L^* g(x) dx \quad (\text{A.1})$$

Definition: L is *formally self-adjoint* if $L = L^*$.

Definition: L^* differential operator is *adjoint* to L if the associated differential equations of L has homogeneous boundary conditions, that if the eq. (A.1) takes the simple form

$$\int f^*(x) Lg(x) dx = \int f(x) L^* g(x) dx. \quad (\text{A.2})$$

or using the inner-product notation

$$(f, g) = \int f(x) * g(x) dx \quad (\text{A.3})$$

eq. (A.2) takes the form

$$(Lf, g) = (f, L^*g). \quad (\text{A.4})$$

The key step to achieve the adjointness is to recognize the importance of elimination of the boundary terms in eq. (A.1). The very same idea leads us to the Green's function (Greenberg, 1971; pp. 22-26). If we find a G function for a given g , for which

$$L^*(G) = \delta(x'-x) \quad (\text{A.5})$$

$$G(a, x) = G(x', b) = 0 \quad (\text{A.6})$$

where δ is the Dirac delta function, then the solution of the equation $L(y)=g(x)$ can be written in the form

$$y(x) = \int G(x', x) * g(x') dx' \quad (A.7)$$

To illustrate the meaning of this seemingly pure formal trick we can imagine an arbitrary physical problem associated with a differential equation, e.g. eq. (2.12), where $g(x)$ is a forcing term on the otherwise autonomous system, represented by the homogeneous $L[y] = 0$ equation. We can realize that

$$G(x', x) * g(x') dx' \quad (A.8)$$

represents a local concentrated influence of the forcing term. Consequently, the right hand side, the integral part, of eq. (2.21) represents the superposition of the localized/quantized influences. That is why the Green's function is often called the influence function.

Appendix B: Minimal Requirements for Quantum Mechanical Observables

Having now provided a generic quantum theoretical framework, two questions naturally emerge:

- How can an observable be conserved (i.e., be a dynamical invariant)?
- How can a conserved quantity be found?

To provide the fundamental ideas for answering the first question, a simplifying assumption is needed. Assume a time independent Hamiltonian H . Let F be an observable in the state ψ . If its value, $\langle F \rangle$ conserved, that is constant, then its time derivative

$$\frac{d}{dt} \langle F \rangle = \frac{d}{dt} \int \psi^* F \psi dx = \int \frac{d\psi^*}{dt} F \psi dx + \int \psi^* F \frac{d\psi}{dt} dx. \quad (B.1)$$

should be equal to zero. Using the complex conjugate Schrödinger equation

$$-i\hbar \frac{d\psi^*}{dt} = (H\psi)^* = \psi^* H. \quad (B.2)$$

eq. (A.1) takes the form

$$\frac{d}{dt} \langle F \rangle = \frac{i}{\hbar} \int \psi^* (HF - FH) \psi dx. \quad (B.3)$$

The integral is vanishing if and only if the commutator of H and F , $HF - FH = [H, F]$, is vanishing,²² that is

$$[HF - FH] = 0 \rightarrow \frac{d}{dt} \langle F \rangle = 0. \quad (B.4)$$

The vanishing of the commutator was trivially true for the case when for the observable operator F was the complex conjugate of the Hamiltonian, $F = H^*$. The vanishing of the commutator obviously provides us a less strict requirement for the observable, but it still requires the full knowledge of H .

Regarding the second question concerning the discovery of conserved quantities, one can further weaken the required conditions as follows:

The solution for the second issue is implicit in the first problem. Namely, if we have an operator U which commutes with H and is invertible, then

$$HU - UH = 0 \rightarrow HU = UH \rightarrow H = U^{-1}HU. \quad (B.5)$$

If U is time independent then eq. (B.5) shows that U is a symmetry operator of the Schrödinger equation.

Definition: U is a symmetry operation of a differential operator L if for any ψ solution of $L U \psi$ is also a solution of L .

For the Schrödinger equation if U is a symmetry operator and ψ is a solution then

$$i\hbar \frac{d(U\psi)}{dt} = HU\psi. \quad (B.6)$$

For U is not time dependent,

$$i\hbar \frac{d\psi}{dt} = U^{-1}HU\psi, \quad (B.7)$$

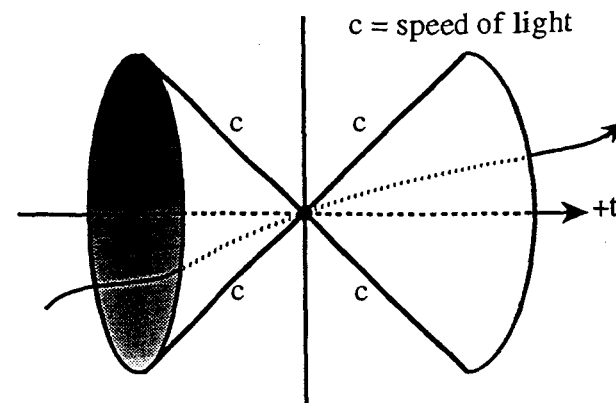
There is, however, an additional physical requirement for the U transformation. In our conceptual framework this means that the inner product invariance postulate is an intrinsic requirement for quantum mechanics. The U transformation is admissible if the normalization of the wave function is not changing with the application of U , that is if

²² If H and G commute then we can choose the eigenfunctions that they will be common eigenfunctions of H and G
 $H\psi = E\psi$
 $G\psi = f\psi$

$$\int \psi^* \psi dx = \int (U\psi)^* U\psi dx = \int \psi^* U^* U \psi dx. \quad (\text{B.8})$$

It follows that $U^*U = UU^* = I$, meaning U has to be a unitary transformation. Clearly, the unitary transformations and also the antiunitary (see the time reversal transformation below) transformation play important role in our theoretical analysis due to the inner product invariance postulate.²³

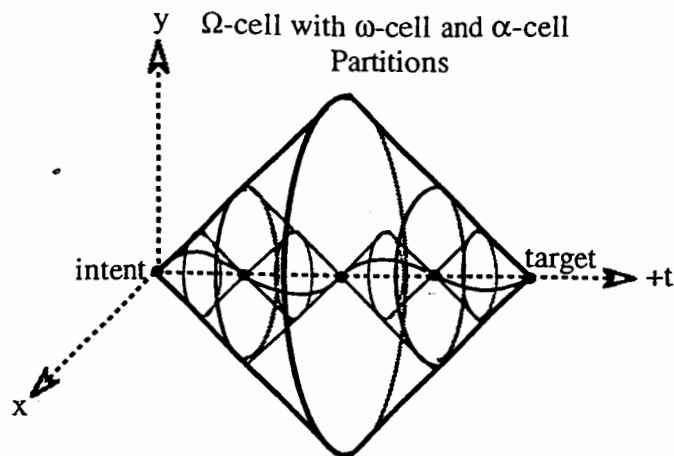
²³ The complex rotation $U = e^{ieF}$ provides us an interesting connection between the certain unitary and Hermitian transformations. The operator F is called the generator of U and it is the observable connected to U if U is not Hermitian. H. Weyl (The theory of groups and quantum mechanics, Dove, New York, 1950, pp. 100,214) considered this kind of rotation transformations while investigating the electric charge $\langle Q \rangle$, as a conserved quantity. This type of transformations are called gauge-transformation of the first kind. Gauge invariance, in quantum mechanics, means that the gauge transformation of a solution would be another solution of the Schrödinger equation.



A Minkowski Light Cone

Figure 1: A Minkowski Light Cone. An unbounded worldline passes through an origin.

Imagine all possible worldlines converging on the origin from the past that might be occupied by an actor. These are all the events that the actor might have perceived as well as all the past events that might have causally affected him. Also, imagine all the worldlines diverging from the origin toward the future. These are all the possible events that may originate from the actor—actions or information—to affect future events.



Note: The goalpath corresponds to a point on an object rotating through one 720° Ω -period, or two 360° ω -periods, or four 180° α -periods.

Figure 2: An Ω -cell Geometry for a Goal-directed Action. Here we see that the worldline segment representing a goalpath is bounded by the point of intent and the target point. In between these endpoints are other points, called *choice-points*, at which sub-goals for subordinate actions are determined. The four ballistic half-turns of the turntable are represented as the points parsing the sinusoidal curve generated over space-time by the rotation event. (The accelerations and decelerations are not depicted).

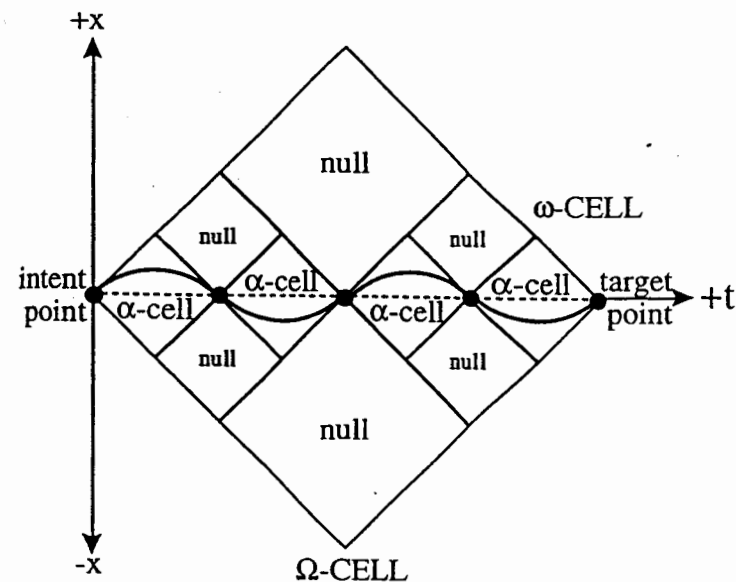
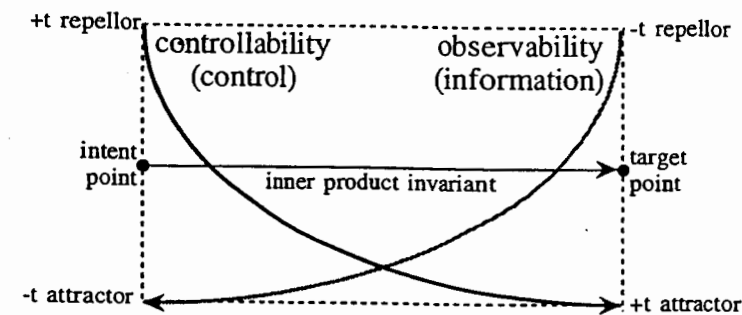


Figure 3: A Schematic Ω -cell Showing its Nested Partitions.



Schematic Representation of Temporal Self-adjointness of Information and Control over Goalpath

Figure 4: Temporal Self-adjointness of the Information and Control. The dual paths denote the complementary conjugate values of information and control at each point along a goalpath traversed by a perceiving-acting system. There exists an inner-product invariant so that the *generalized action* quantity (defined as the inverse flow of information and control) is conserved. As one quantity increases, the other decreases, so that the bi-temporal integrals always sum to yield the same total amount of generalized action over a given goalpath. If the path is not a goalpath, then this quantity will not be conserved.

Organism Perspective
(Action)

Environment Perspective
(Perception)

Energy (control)(+t)

$$y(t) = k x(t) + \int_0^t K_o(t, s) x(s) ds \quad \text{Equation I}$$

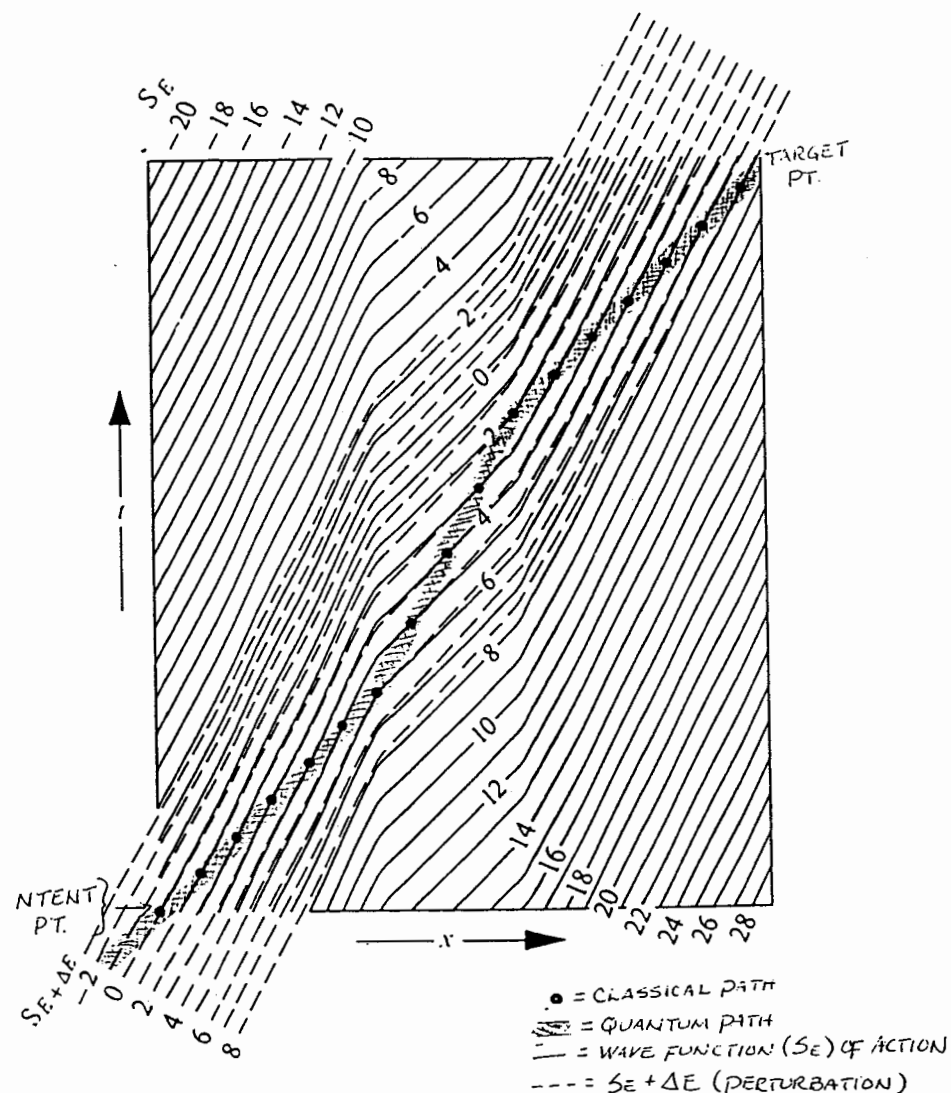
$$x(t) = k y(t) + \int_0^t K_E(t, s) y(s) ds \quad \text{Equation II}$$

Information (detection)(-t)

$$x^*(t) = k y^*(t) + \int_0^t K_o^*(s, t) y^*(s) ds \quad \text{Equation III}$$

$$y^*(t) = k x^*(t) + \int_0^t K_E^*(s, t) x^*(s) ds \quad \text{Equation IV}$$

Table I: The Integral Equations Representing the Perceiving-acting cycle. This system of adjoint equations are the solutions to the differential equations discussed in Section 2 (See Shaw & Alley, 1985, for discussion).



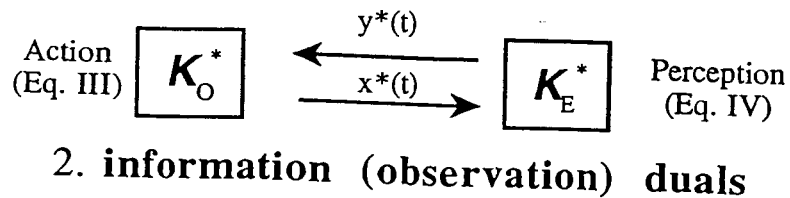
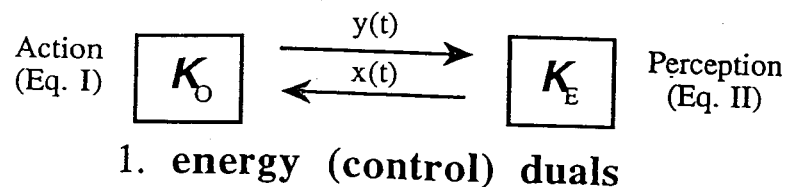


Table II: The Adjoint Operators Representation of the Perceiving-acting Cycle. Note the correspondence to Table I.

Figure 5: Emergence of the 'Knowledge Wave' within the Ω -cell. (INSERT HERE).

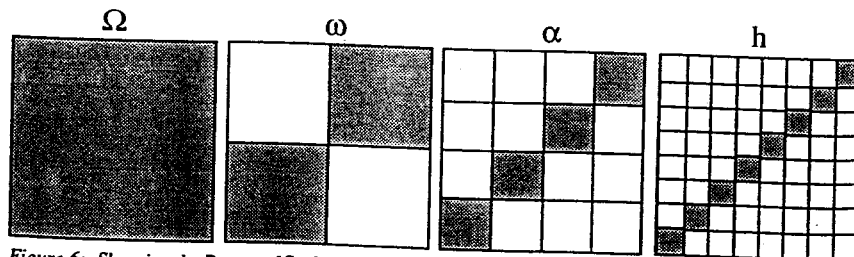


Figure 6: Showing the Range of Scales for the Weighing Function in the Feynman Path Integral.

KIND OF LAW	FORMULATION
I. Classical mechanics	event ₁ —(mechanical law)—>event ₂
II. Classical psychology	stimulus—(association law)—>response
III. Quantum mechanics	observation ₁ —(QM law)—>observation ₂
IV. Intentional dynamics	goal-info.—(effectivity)—>goalpath info.

Table III: A Comparison of Law Forms