Appendix 1: General Negative Feedback Loops

There are many forms of negative feedback loop, not all of which are control systems. They are introduced here to put the simple control system into a context that we will use when we deal with side-effect loops and similar configurations.

To begin with, we need a definition of "negative feedback", since it is not the same as when someone says that a teacher gave a bad student negative feedback. As used in PCT (and in engineering), *negative feedback exists when a change in some property results in an influence that opposes the change*.

The simplest illustration of negative feedback may be the spring of Figure A1.1. In the left ("before") panel, the spring is stretched by a force due to a mass hung on its end. The gravitational force is balanced by a Hooke's Law force that would recompress the spring if the mass were to be removed. The negative feedback is simply between the applied force and the force due to the energy stored in the spring by the applied force. When the applied force is removed, the mass oscillates up and down until frictional forces have distributed the energy back into the environment of the spring.

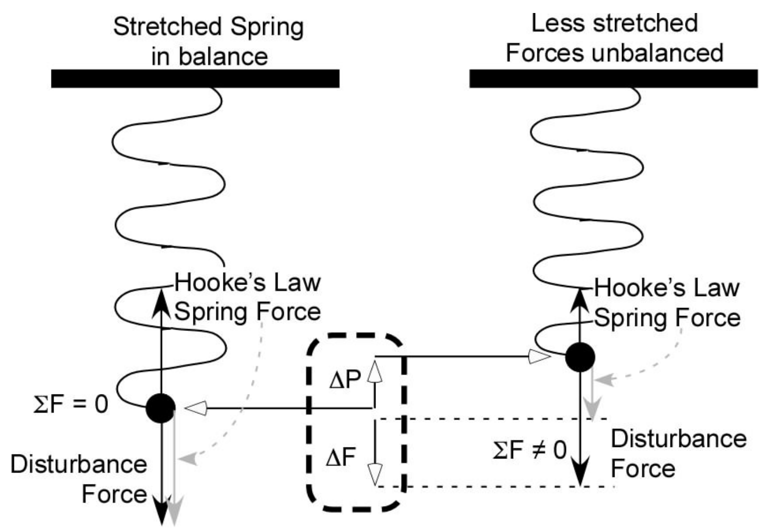

Figure A1.1 A spring stretched by a mass (left panel) and after the position of the mass has been raised by some means not shown (right panel). After the mass has been raised, there is a net force that would again lower the mass. After stabilizing due to frictional losses, the situation would revert to that of the left panel.

"Equilibrium systems" may be unstable, metastable, or stable, depending on whether they gain or lose energy by changing their configuration. The spring is an example of a stable equilibrium system. Other examples are a ball in a bowl, which returns to the bottom of the bowl after being pushed up the side, a pendulum that returns to the vertical after being pulled aside, and so forth. Such a system might be called a "N=1" negative feedback system, because it has only one place where external influences can affect the variables in the system.

Figure A1.2 illustrates a range of negative feedback systems from N=1 to N ≈ ∞, the vortex[1]. A control system is a N=2 system, but not all N=2 systems are control systems. In order for a N=2 system to be a control system, one side must have substantially more amplification than the other. Powers, in a message to the control group mailing list dated 931130.1545 MST, said: "I would consider it control if the (negative) loop gain was greater than 5 or so."

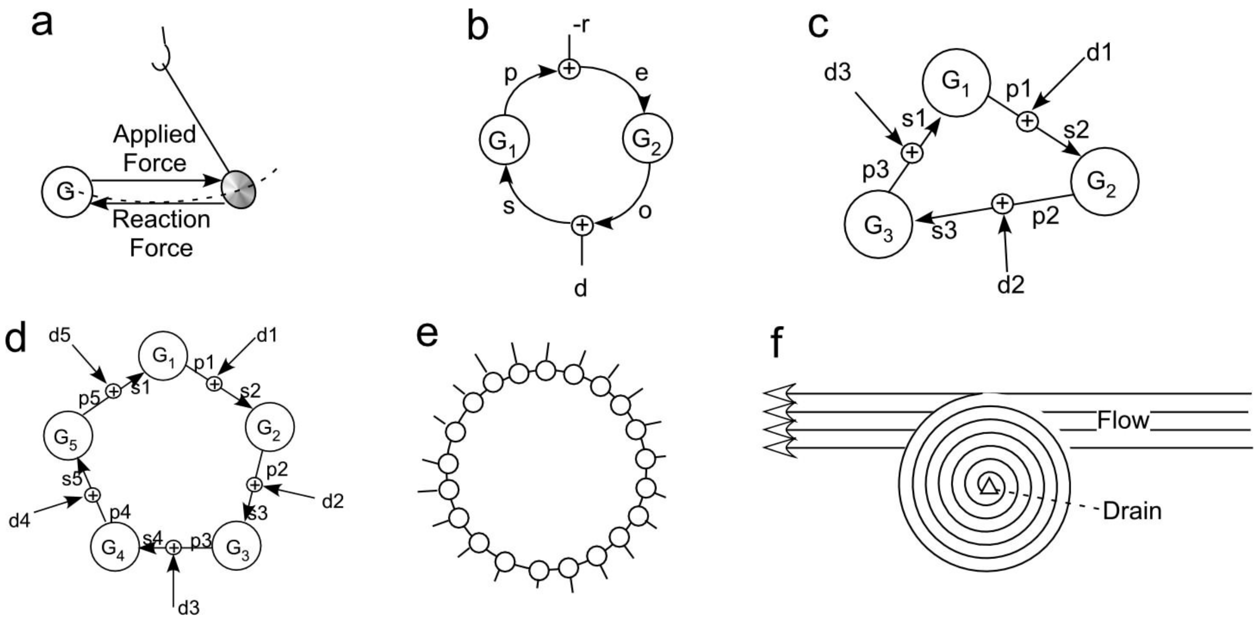

Figure A1.2 A series of forms of negative feedback systems, from (a) the Newtonian reaction provided by gravity to an applied force on a pendulum, through (b) a standard control loop and (c, d, e) a few longer loops to (f) a simple vortex flow in a stream or down a drain. All of these loops show a kind of stability, but only the asymmetry between the gains of G_1 and G_2 in the loop (b) gives true control of a single variable.(In b, the sign reversal that is usually placed in the comparator has been instead placed in the output function. This makes no difference to the function or the mathematical structure of the control loop.) Large circles in the diagrams represent places that may use energy to convert or amplify the effect of influences represented by incoming lines and arrows to create influences on other places. Small circles with a plus sign represent conceptual places where influences from separate sources may add to signals circulating around the loop.

The simple control loop has only two inputs, which we call "reference" and "disturbance" when they have those functions within a PCT hierarchy. If, however, we consider the loop itself, the two inputs are symmetrical. To see this, we can do a simple algebraic analysis of the equilibrium states of the loop, making the simplification that G_1 and G_2 are taken to be simple

1 Of course N is not truly infinite. It is of the order of Avogadro's number, the number of molecules in one mole of a substance, which is around 6×10^{24}, a number large enough to deter consideration of the individuals.

positive multipliers, but the G_2 function reverses the sign of its input, which makes the gain around the loop negative.

In this simplified case[2], $p = G_1 \times s$ and $o = -G_2 \times e$.

$p = G_1 \times s = G_1 \times (d+o) = G_1 \times (d-G_2 \times e) = G_1 \times (d-G_2 \times (p-r)) = G_1 \times d - G_1 \times G_2 \times p + G_1 \times G_2 \times r$

Collecting the terms in p, we have

$p \times (1+G_1 \times G_2) = G_1 \times d + G_1 \times G_2 \times r$

which gives

$p = r \times G_1 \times G_2/(1+G_1 \times G_2) + d \times G_1/(1+G_1 \times G_2)$

When $G_1 \times G_2 >> 1$, p is nearly equal to r, provided $G_2 >> 1$. Since we will be interested in the symmetrical case when we consider the longer loops, we can set $G_1 = G_2$ and $G = G_1 \times G_2$ (the loop gain). The above equation can be rewritten as:

$p = r \times G/(1+G) + d \times G/((1+G) \times G_2)$

Writing $G/(1+G) = U$ to simplify the visual appearance of the equation, U will be near but below 1.0 if the loop gain G is high. We have

$p = r \times U + d \times U/G_2$

The point of this exercise is to show that when $G1 \times G2 >> 1$, p is very responsive to changes in r, but when G_2 (called the “output function” in a control loop) is appreciably greater than unity, p is much less responsive to changes in d. The same analysis, starting with o instead of p, leads to

$o = -d \times U - r \times U/G_1$

In words, the “p-type” variable in the control loop is most responsive to variations in the value of the immediately following “d-type” input.

This fact holds for the longer loops, as can be seen by using the same kind of analysis of the signals around the N=3 loop of Figure A1.2c, starting with p1. If all three gains are the same and equal to M, we have

$p1 = -d1 \times M^3/(1+M^3) + d2 \times M^2/(1+M^3) - d3 \times M/(1+M^3)$

If $M>>1$,

$p1 \approx -d1 + d2/M - d3/M^2$

2 This simplification yields a stable state that is achieved by a real system only when the variables change infinitely slowly, but if the functions in the system are linear, the system dynamics can be determined by treating all the symbols as the Laplace Transforms of the signals and functions. The simplification gives approximate answers when the variables change slowly, and is easy to follow.

Again, p1 is most responsive to variations in d1, the immediately following input, next most responsive to d2, the next one along, and least responsive to d3, the one immediately preceding it. The same kind of static analysis leads to the same conclusion for all the other loops. For example, the N=5 loop in Figure A1.2d gives the result, if all the gains are equal to M and M >>1,

$p1 \approx -d1 + d2/M - d3/M^2 + d4/M^3 - d5/M^4$

which, if M is large enough, is almost the same as

$p1 \approx -d1 + d2/M$

no matter how long the loop.

In the vortex, which is in effect the end-point of increasing the number of stages without limit, the localization of influence is shown by the distance around the vortex over which the influence of the introduced stick can be seen. In a slow-moving stream or one with a tiny drain, the stick might disrupt the vortex entirely. In a very fast stream or a large drain, other possibly nonlinear fluid dynamical considerations come into play and the analysis is inapplicable, but for an intermediate flow, most of the circuit of the vortex is not visibly affected by the introduction of a small enough stick. The effect is localized.

In the diagrams of Figure A1.2, the kind of influence represented by one line has no necessary relation to the kind of influence represented by another, except that a "d" line must be of the same kind as the "p" line to which it is added. For example a "G" might have a stream of photons as its "s" input and produce a mechanical force as its "p" output. Of course, this implies that the preceding "G" must output a photon stream, but somewhere else in the loop the influence might be a disturbance to someone's perception of social propriety and the output whatever pattern of actions that person uses to counter the disturbance, such as, possibly waving a placard.

As is often the case in systems with many components, the analysis is relatively easy when the situation has obvious symmetry, as do the simple rings in Figure A1.2. In real life, such rings are very likely to have cross-links, which destroy the symmetry. In such circumstances, it is usually true that an exact analysis is feasible for small numbers of components, and that a close approximation can be made by lumping large numbers of components together, as is done in a "mean field" type of analysis. In the middle, however, there is a range of complexity that allows neither a statistical combination nor an exact analysis. It is in this area that most research needs to be done.