CYCLES OF CONFLICT

A Mathematical Model for Collins's Theory of Conflict Escalation*

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ABSTRACT

The theory of conflict escalation offered by Randall Collins (2012) is a noteworthy achievement. Engaging critical issues of violent conflict, Collins presents a compellingly plausible theoretical description based on his extensive empirical research and simultaneously sets a new challenge for sociology: explaining the time dynamics of social interaction. However, despite heavy reliance on the mathematical concept of positive feedback loops in his theory, Collins does not present any mathematical specification of the relationships among his variables. This paper seeks to fill that gap by presenting a mathematical model that can parsimoniously account for many features of Collins's theory. My model uses perceptual control theory (PCT) to create multi-agent simulations of the time dynamics of conflict that mirror and extend Collins's verbal propositions. This reformulation of the theory improves its conceptual clarity, reveals dynamic aspects of conflict not obvious in the verbal formulation, and gives the theory more wide-ranging generalizability.

CYCLES OF CONFLICT:

A MATHEMATICAL MODEL FOR COLLINS'S THEORY OF CONFLICT ESCALATION

In his 2011 Presidential Address to the American Sociological Association, Randall Collins has offered a new theory of the dynamics of group conflicts (2012). Building on his own extensive investigations of episodes of interpersonal violence and warfare (2008), Collins presents conceptual models of "conflict escalation" and "de-escalation" (in his words, "C-Escalation and D-Escalation") in violent struggles between opponents (2012:4, 11). This new theory represents a significant achievement in several respects. In a world fraught with internecine conflicts between ethnic factions, omnipresent threats of terrorism, and governments rendered dysfunctional by deadlocked ideological battles, the topic itself is a crucially important one for sociology.

Furthermore, his new theory is supported by substantial empirical data. In devising the theory, Collins has drawn upon his own extensive research into incidents of violence and social conflict, including investigations into the "the micro-sociology of violence," which have involved analysis of data gathered by a variety of empirical methods, including the collection of "photos and videos, . . . participants' detailed accounts, ethnographic observations, forensic reconstructions (*e.g.*, bullet paths and number of shots fired), data on bodily physiology, and subjective phenomenology" (2012:4), all of which he describes in considerable detail in a recent book (2008; see also Collins 2009). In addition, his recent research projects have examined processes of victory and defeat in battles (Collins 2010) and displays of ritual solidarity following the 9-11 terrorist attack (2004). In short, his theory rests on a more solid empirical foundation than much of the other contemporary theorizing in sociology.

Collins's new theory also represents a significant achievement in the way that he has implicitly upped the stakes for sociological explanation by focusing on particular incidents of conflict escalation, not just broad social patterns, and thus has taken the time dimension of social interaction seriously. In his paper, Collins portrays the "time dynamics" of conflict as a challenging new area for sociological research (2012:13). With some exceptions,[1] the bulk of the empirical research published in the field of sociology hs relied on cross-sectional analyses of statistical data, in which the time variable is not explicitly considered. The statistical studies that use longitudinal designs to account for changes over time typically analyze cross-sectional relationships between variables at several points in time, rather than examining the ongoing flow of social events. Theorists and researchers in the social micro-interactionist tradition have tended to use either qualitative observation methods, in which time is considered only implicitly, or laboratory experiments that probe relationships between variables without investigating changes over time. It has mostly been left to historically oriented sociologists to focus on issues of time, but their main analytical techniques have usually involved comparison of different societies at different times, rather than a close examination of the unfolding of particular historical events (but see Sewell 2005). Thus, time, as a continuous variable, has been largely neglected by sociologists, and Collins, to his credit, has raised that issue.

Another way in which Collins's new theory is noteworthy is in his recourse to the language of systems theory, an approach that was regarded sixty years ago as a promising new development in sociological thinking, but has since largely faded from popularity among sociologists, for reasons not necessarily connected to the empirical or theoretical value of the perspective.[2] Collins makes use of the systems-theory idea of positive feedback loops involving the variables in his model, so that increases in group solidarity and ideological polarization, for instance, are modeled as leading to increases in the intensity of conflicts between groups, which then increases the solidarity and polarization of the group. Thus, conflict is seen in his model as likely to result in the kind of runaway intensification that characterizes arms races and other conflict spirals, although Collins's model of de-escalation presents factors that may lead to diminished group solidarity and, thus, the diminution of conflicts. Systems models, with their emphasis on feedback loops, are particularly useful for understanding the behavior of systems of variables in which reciprocal causation occurs, so that changes in one variable affect another and vice versa, as when escalation by both sides in a conflict provokes answering responses for both. The more conventional causal modeling approach, still used in most statistical research (as well as most theoretical thinking) in sociology, is much less well suited for handling reciprocal causation, so Collins's invocation of systems thinking makes sense as a way to describe the incidents of conflict escalation that are his primary concern.

Yet another advantage of framing a model in terms of systems thinking is that systems theory has a well-developed mathematical base, which raises the possibility of constructing a mathematical model and using it to do computer simulations of the theory. From his writings, it appears that Collins has long been intrigued by just this possibility. For instance, in discussing the positive feedback loops in his model of conflict escalation, he remarks, "Notice that all feedback loops in the model are positive. If we were to do a computer simulation, conflict would escalate to infinity" (2012:10). In an earlier book, Collins (1992) devoted a final chapter (called "Can Sociology Create an Artificial Intelligence?") to imagining the possibility of representing one of his earlier theoretical models in terms of a computer simulation. Despite his apparent interest in simulations, however, Collins has not yet taken the step of putting any of his models into a mathematical form.[3] The purpose of this paper is to explore precisely that possibility by constructing a mathematical model of conflict based on Collins's model.

In this paper, I offer a multi-agent mathematical model of dynamic social interaction that shows how the conflict processes described by Collins can be represented as computer simulations. Although my model makes use of systems theory, the simulated agents in the model use negative feedback loops, rather than positive feedback loops. As we will see, collective processes of positive feedback, just like those in Collins's model of conflict escalation, can emerge from the interactions of these simulated agents. My analysis will rely on perceptual control theory (PCT), a psychological model of neural circuits in the human brain, which proposes that brain circuits are organized into hierarchies of negative-feedback control systems, and that individuals use these control systems to manipulate their own environments with the purpose of controlling their flow of perceptual input in accordance with internally generated preferences and expectations (Powers [1973] 2005, 2008). This systems-theory paradigm has already served as a conceptual basis for a considerable body of research in sociology and social psychology (see McClelland and Fararo 2006, Robinson 2007), and the modeling results presented here are extensions of simulations reported by McClelland (2004, 2006).

My paper has four sections, beginning with an overview of Collins's theory of conflict escalation and de-escalation and a critique of its conceptual adequacy for reformulation as a mathematical model. The second section describes the mathematical model used for my simulations, in the context of the theoretical perspective from which the model is derived. The third section reports the results of these simulations and discusses how they sometimes extend the theoretical conclusions drawn by Collins or suggest modifications of them. The paper ends with an evaluation and discussion of what has been gained by using multi-agent mathematical models to supplement and reformulate a theory presented originally in the familiar form of verbal propositions and schematic diagrams.

COLLINS'S MODEL OF CONFLICT

Overview of His Model of Conflict Escalation

Positive feedback loops abound in Collins's theory of conflict escalation (2012). He illustrates his theory with seven diagrams that display hypothesized relationships between variables, all featuring one or more feedback loops; his first diagram shows only two variables, and his most complicated diagram contains twelve interrelated variables. One can get a sense his core theory by looking at Figure 1, which reproduces the third in the series of diagrams offered by Collins.

[Figure 1 about here.]

The diagram displayed in Figure 1, which Collins labels, “Escalating Conflict: Atrocities and Polarization,” presents the relationships between four key variables in his model. Straight-line arrows in the diagram indicate relationships assumed to be positive, and the curved arrows show the completion of positive-feedback loops. Taking, for instance, the two central variables in his model, Collins argues, “external conflict increases group solidarity. . . . [b]ut solidarity also causes conflict.” Similarly, he sees the occurrence of atrocities, defined as “opponents’ actions that we perceive as especially hurtful and evil” (2012:12), as leading to ideological polarization between the two sides, which in turn intensifies the conflict and leads to yet more atrocities (2012:3-5).

In his more complex diagrams, Collins displays this core set of variables twice, once for each side of a two-party conflict, the two sides linked by a double-headed arrow for the variable *conflict,* which appears in each of the core sets of variables. The full model also contains two additional variables for each side, one labeled “mobilizing material resources,” and the other, “seeking allies, forcing out neutrals” (see Collins 2012:9, Figure 7). Causal arrows in these diagrams indicate a positive feedback loop between *mobilizing material resources* and *conflict,* and a second positive feedback loop that links *seeking allies, forcing out neutrals* to *conflict* via *mobilizing material resources* as an intermediate variable.

Need for Conceptual Clarification

This model, as presented, does not immediately lend itself to mathematical reformulation as a multi-agent model. While the model has considerable intuitive plausibility, Collins has relied heavily on common-sense definitions of key variables in his model, definitions that may not always meet the test of mathematical rigor. For example, the variable *conflict,* as we see from Figure 1, is central to his model, but he offers this variable without defining it, other than by referring to the works of such venerable theorists as Simmel ([1908] 1955), and Coser (1956). While the work of these sociological pioneers offers many subtle insights about human conflict, they themselves had no ambition to render their theories as mathematical models, instead relying implicitly on common-sense understandings of what conflict means, and the diagrams offered by Collins do little to clarify his definition of conflict.

The variable *solidarity* presents similar problems for the analyst seeking to construct a mathematical representation of Collins's model. In his use of solidarity as a variable, just as with conflict, Collins has drawn on a long tradition in sociology, dating in this case back to Durkheim ([1893] 1933, [1897] 1951), but this tradition gives scant guidance for the purpose of constructing a mathematical specification. Solidarity is the attribute of a group of actors forming one side of a dispute, but does their solidarity consist of unity of purpose, emotional adherence to the shared cause, or unified actions against the opponent? Thus, is he talking about similarity in perceptions, in emotions, in actions, or all three? These ambiguities in the traditional definition of solidarity in sociology make the concept somewhat problematic for formulating a mathematically based theory.

Other key variables in Collins's model also suffer from a lack of conceptual precision. Collins explicitly defines the variable *atrocities* as a matter of the perceptions of people on one side of the conflict interaction: "Atrocities are opponents' actions that we perceive as especially hurtful and evil . . ." (2012:2). However, he does not offer any operational definition that would enable other observers to judge whether an atrocity has been committed.

He provides no definition at all for the variable *ideological polarization,* and, although his more elaborate schematic models do not contain a double-headed arrow linking the dual appearances of this variable in the core models for the two sides of the conflict, the sense in which this variable can be conceptualized as applying to one side of a dispute separately from the other is not entirely clear. In its common-sense meaning, polarization would seem to refer to a gulf in perceptions between the two sides of an interaction, not just the thinking of one side or the other, although perhaps by polarization Collins instead means the extent to which each side takes an extreme position along some ideologically defined continuum.

In sum, by constructed his theory primarily in verbal terms, Collins has relied for his definitions of key variables on a sociological tradition of theorizing that emerged from common-sense discourse about social problems, and the resulting definitional ambiguity presents some problems for constructing a more tightly defined mathematical model based on his theory. Hence, one must make some arbitrary assumptions about Collins's implicit definitions to render his model in mathematical terms, and the model, when transformed mathematically, may not necessarily look anything like his diagrams. Nevertheless, I will argue that the multi-agent mathematical model that I offer covers much the same ground substantively as his theory and that several of his key variables can be subsumed into the parameters of my model.

THE PERCEPTUAL CONTROL THEORY MODEL

The Basic Model

Perceptual control theory (PCT), the computational modeling approach that I use in this study, offers a conceptual model of an intentional human actor, capable simultaneously of rational calculation and emotional response. The mathematical realization of the model is derived from control-system engineering, and the model portrays the neural organization of the human brain as a complex arrangement of nested layers of negative-feedback control loops. This neuropsychological theory of behavior developed was developed by William T. Powers ([1973] 2005, 2008), a control-systems engineer rather than a psychologist by training, but the theory has found applications in a wide variety of psychological research areas, ranging from animal behavior studies (Bell and Pellis 2011; Pellis, Gray and Cade 2009; Pellis and Bell 2011) to clinical psychology (Mansell and Carey 2009; Mansell, Carey and Tai 2012). The theory has also provided the inspiration for two prominent research programs in interactional sociology, affect control theory (Heise 2007) and identity control theory (Burke and Stets 2009), as well as for research in collective behavior (McPhail 1991). With its basis in systems thinking, the PCT modeling approach seems especially useful for examining Collins's theory of conflict.

The negative feedback loop central to the PCT model describes a behavioral process that allows humans and other animals to maintain control of their own environments by acting to reduce discrepancies between their internal preferences and expectations and their perceptions of what is occurring around them. The core idea of this approach is that organisms seek to control their own environments by controlling their perceptions of those environments. The theory further hypothesizes that a hierarchical structure of nested control systems in the human brain allows for the simultaneous control many kinds of perceptions, from concrete perceptions of the physical world surrounding us, to rational perceptions such as logical categories and programs of action, and ultimately to highly abstract perceptions of human values and of personal and group identity (see Powers [1973] 2005, 2008; McClelland 1994).

Without going deeply into details of the theory, we should also note that conflict has a significant place in the model, as we will see when the model is directly applied to Collins's theory. Conflict is expected to occur whenever two control systems operating in the same environment have incompatible goals, and conflict can occur either between two control systems in the same brain, as when a person dithers between two equally attractive options, or between the control systems of two individuals, as when two people sharing the same environment don't share the same goals. This second type of conflict—social conflict between interacting individuals—is the type my models explore.

Figure 2 shows a schematic diagram of the negative feedback loop that lies at the core of the PCT model. In the figure, the area with the gray-shaded background represents the brain and nervous system of a person, while the area outside the gray shading represents the person's environment. The segment of the loop that lies within the person's body corresponds roughly to a "reflex arc" as conventionally understood, with *perceptual input* from sense organs, then information processing in the brain, in which the input is *compared* to memories that serve as *reference* values for these perceptions, and finally motor *output*, as discrepancies between the perceived input and the reference values produce *error signals* that activate the muscles involved in physical responses.

[Figure 2 about here.]

The lower half of Figure 2 depicts the segment of the negative feedback loop that passes through the organism's environment. The diagram shows how a person's physical actions have *feedback* effects on perceptions. Physical *actions* affect variables in the environment that are the sources of the person's perceptions. Because these physical actions are driven by error signals emerging from a comparison function in the brain, effects of these actions tend to be compensatory, reducing discrepancies between the person's perceptions and expectations for those perceptions. When other factors in the environment, labeled *disturbances* in the diagram, affect the variables perceived, the person's actions tend to reduce the impact of disturbances, thus keeping the variables in the environment, as well as the person's own perceptions of these variables, in *control* by eliminating much of the variation that would otherwise occur. Of course, a person's physical actions may also have *unintended effects* on other variables in the environment besides those that are controlled.

One can demonstrate by mathematical analysis that the variables controlled by the actions of this negative feedback loop are *input* variables—perceptual signals—rather than the output variables, that is, the organism's physical actions, which must be free to fluctuate to counter the effects of disturbances (see Powers [1973] 2005). Whenever the organism's perceptions are reasonably accurate, successful control of perceptual variables results in a corresponding stabilization of environmental variables. Thus, the operation of the loop stabilizes the organism's flow of perceptual input to match its reference conditions by stabilizing the aspects of its environment that its perceptions represent. From this perspective, the physical actions of an organism in response to any particular stimulus in its environment may vary, depending on its own internal reference conditions and the presence or absence of other disturbing factors in its environment.

A PCT Model of Conflictive Interaction

The PCT model to be offered in this study as a possible mathematical specification for Collins's theory of conflict (2012) comprises four simulated "agents," each modeled as negative-feedback control system, all controlling the same variable in a shared environment. Figure 3 presents a schematic diagram of the model. All four simulated agents in the model have the same mathematical structure, a control-system model of the simplest kind: one level of control of a one-dimensional variable. In Figure 3 each of the gray-shaded rectangles represents the "brain" of one agent, and examination of Figure 3 in comparison to Figure 2 reveals that all the agents have the same interior components as the control-system model in Figure 2, but with the orientation of components rotated 90 degrees to the left for Agents 1 and 2 and 90 degrees to the right for Agents 3 and 4. In the simulations of conflict to be presented, Agents 1 and 2 will represent one side of the conflict and Agents 3 and 4, the other.

[Figure 3 about here.]

The interactions between the two opposing sides, and also the interactions between agents on the same side of the conflict, are modeled as mediated entirely by their joint attempts to control a single variable in their shared environment. This variable represents the "stakes" of the conflict—the focus of the fighting—and we might think of it as representing money or territory or comparative prestige. From the PCT perspective, conflict occurs whenever the two sides use different reference standards in their attempts to control an environmental variable, and victory or defeat in the contest is represented by the extent to which side or the other succeeds in bringing the variable into line with its own preferred reference conditions.

Compared to the human protagonists portrayed in Collins's theory, these simulated agents are radically simplified. They have neither behavioral mechanisms for communicating with each other nor any ability to monitor the actions of agents on their own side or the other, except insofar as they perceive the impact of such actions on the environmental variable on which their attention is fixed. These models do not provide the complexity needed for simulating any of the higher mental powers, such as those involved in forming a perception of personal or group identity or in perceiving the actions of the other side to be an atrocity. Clearly, with such simple models one cannot expect to reproduce the subtlety or sophistication of Collins's analysis.

Nevertheless, as I will show, this drastically simplified model of interaction is sufficient to generate patterns characteristic of the basic dynamics of conflict described by Collins and, in particular, the apparent pattern of positive feedback in interactions of conflict. Empirical research has demonstrated that even very simple control-system models can explain patterns of behavior involving perceptions as complex as coordination of movements by groups of individuals across a field (McPhail and Wohlstein 1986) or maintenance of a sense of personal identity (Burke and Reitzes 1991). And large body of research has shown that the moderately complex actions and perceptions involved in tracking experiments—as for example when a subject uses a mouse to pursue a target on a computer screen despite disturbances—can be predicted with remarkable precision (r^2 often in excess of .98) using simple control-system models (Bourbon 1990, Bourbon et al. 1990; Marken 1980, 1985, 1988; Powers 1978; McPhail and Schweingruber 2006). In principle, it may be possible to construct more complex PCT models, capable of simulating a greater proportion of the kinds of sophisticated behaviors that Collins is describing, but even simple models can capture recognizable patterns of dynamic behavior.

Each individual agent in these simulations is described by the following mathematical model of a negative feedback control system with a total of seven variables.[4]

p The *perceptual input signal* for the control system. For these simulations the perceptual signal is always set equal to the value of the environmental variable, as if the agent possesses perfect perception of conditions in its environment. In other words, the input function for each control system (see Figure 2) has been modeled simply as an identity function.

r The *reference signal* for the control system. In my simulations of conflict, the reference signals for agents on Side 1 are set to positive values (or zero), while reference signals for agents on Side 2 are set to negative values (or zero).

o The *output signal* for the control system, a function of the difference between *r* and *p*. In these simulations the output action for the agent is taken as exactly equal to the output signal, as if the agent's body were perfectly efficient in translating neural impulses into muscular responses. Thus, the output function for each control system, like the input function, has been modeled as an identity function.

v The value of an *environmental variable*, which is affected by the combined output of the four agents, as well as by the disturbance *d* (defined below). In my simulations of conflict, I will refer to this variable as the *contested variable.* Like the input and output functions, all of the feedback functions have been modeled as identity functions in order to simplify the model, as if these agents were perfectly efficient in translating their actions into physical impact on the environmental variable.

d The *disturbance* acting on the environmental variable, that is, the sum of all other environmental forces affecting *v*, net of the outputs of the four control systems. For the purposes of the simulations reported here, the disturbance variable is set to a constant value of zero, though it could also be modeled as a vector of values that change over time.

g The *loop gain* of the control system, which is proportional to the speed at which the control system corrects its errors and also corresponds to the precision with which the control system matches its perception to its reference signal.

s A constant *slowing factor* (also described as a "leak") introduced to allow the accurate representation of a continuous (analog) process in the form of a discrete (digital) simulation. In all of these simulations, $s = 0.0025$.

These mathematical simulations are iterative, and the changing values of the system of variables are recalculated at each iteration over a "run" of 100 equal-time intervals from $t = 0$ to $t = 100$. Some of the variables described above, including the reference signal, the disturbance, the loop gain, and the slowing factor, are *parameters* of the model; they are either constant values or vectors of changing values that are set arbitrarily in advance of a given run of the simulation.[4] By adjusting these parameters from run to run, the analyst can investigate the behavior of the simulation model in terms of the dynamically changing values of the remaining variables, including the perceptual signal, the output signal, and the environmental variable, which are recalculated at each iteration. Formulas for each of the recalculated variables for agent k at time t are as follows, where o_{kt} is the output value, p_{kt} is the perceptual signal, and r_{kt} is the reference signal:

$$p_{kt} = v_{t-1}$$

$$o_{kt} = o_{kt-1} + s\ \{g\ (r_{kt} - p_{kt}) - o_{kt-1}\}$$

$$v_{kt} = o_{1t} + o_{2t} + o_{3t} + o_{4t} + d_t$$

None of the variables in this simulation model corresponds precisely to the variables in Collins's model of conflict, and, as I have argued above, the Collins has not always defined his variables precisely enough for the purposes of constructing a mathematical model. To fit my simulation model into the framework proposed by Collins requires some creative use of parameters in the PCT model, which in the context of the model are offered as having analogous functions to the variables of his model. Here are my definitions for some core variables in Collin's model:

Conflict: In models of control-system interactions, conflict is not a variable but rather an interactive outcome that inevitably occurs when two or more control systems attempt to control the same environmental variable using different reference standards. In his recent article on "the micro-sociology of violence" Collins (2009) argues that violence can result when "individuals (or groups) confront one another at cross purposes" (p. 571). His description of conflict as "action at cross purposes" (p. 569) fits well with the definition of conflict that I am using in these simulations. When such conflicts occur, the dynamic behavior of these models is such that the output of the two interacting systems begins to diverge, with one system, in effect, pulling in one direction to bring the environmental variable into line with its preferences, while the other system pulls in the other (see McClelland 2004, 2006). In this context, I will define as a variable the *intensity* of the conflict, measured by the degree of divergence in system outputs.

Solidarity: This variable has been usually taken to refer to feelings of unity or agreement within a group, as well as the willingness of group members to take action in support of the group. Collins, as I have noted, leaves the definition of solidarity implicit in his model, although he notes that in conflict situations solidarity "makes one willing to sacrifice oneself for the group" (2012:2). While a definition in terms of self-sacrifice is obviously well beyond the scope of my rudimentary simulation model, a definition of solidarity in terms of agreement between agents is possible. For purposes of these simulations, the *solidarity* of the agents on one side of the conflict in these simulations will be defined as the extent of agreement of their reference signals. Furthermore, if the agents have similar reference signals, we will say that solidarity increases as the loop gain of the lower-gain agent increases to match that of the higher-gain agent, which might be taken as an increase in the agent's willingness to act on behalf of the group.

Polarization: For purposes of these simulations, I will take polarization to refer to the extent of differences in reference values between the two sides, rather than their positions on some ideological continuum.

SIMULATION RESULTS

Conflict

My first objective in presenting results of simulations using the PCT model is to show how this model can generate the positive-feedback loops that in Collins's view are fundamental patterns of conflict. Figure 4 shows the behavior of the PCT model in a conflict situation. In this simulation, the conflict occurs between Agent 1 and Agent 3, with Agents 2 and 4 in effect sitting on the sidelines. The horizontal axis indicates time, and the units of time shown here are the 100 iterations over which the mathematical model is recalculated. The vertical axis indicates units of the *Contested Variable*, a label for the environmental variable that the agents are jointly controlling. Because the input, output, and feedback functions of the control systems have been modeled as identity functions, the vertical axis can also be used to graph the values of the reference signals of the agents, as well as their outputs and the joint impacts of those outputs on the contested variable. In this simulation Agent 1 has been given a loop gain of 40, and Agent 3, a loop gain of 10. Agent 1 has been assigned a reference signal of 20 points in the positive direction, while Agent 3 has a reference of –20. Because Agents 1 and 3 both attempt to control the same variable, but with different reference values, the interaction produces conflict. Agents 2 and 4 are inactive, having been assigned reference values of zero and a loop gain of zero, so that their output values remain at zero throughout the simulation.

[Figure 4 about here.]

The sharp divergence that we see in Figure 4 between the outputs of the two active agents is a characteristic signature of control-system conflict. As one agent pulls in the positive direction, the other pulls in the negative direction, so that the efforts of one agent are largely counteracted by the other. Looking at the first 30 iterations of the simulation, we see that Agent 1, because it has higher loop gain, begins more quickly than Agent 3 to correct the discrepancy that it perceives between the initial value of the contested variable and its preferred reference value; in effect, Agent 1 has gotten the jump on Agent 3 and now has the upper hand in the struggle for control of the contested variable. However, as the gap between the value of the contested variable and Agent 3's preferred reference value grows larger, Agent 3 begins to pull harder in the negative direction, as is shown by the slight downward concavity of the curve for Agent 3's output. At the same time, the gap narrows between the contested variable and Agent 1's reference, so that the rate of increase of Agent 1's output begins to slow down, as is again indicated by the downward concavity of the curve. An important point to note is that a control system's response, in terms of output, is always proportional to the size of its perceived error, so that, as the difference between its perceptual signal and its reference signal decreases, the rate of error correction decreases, as well.

At about iteration 40, interaction of the two agents reaches an equilibrium point, with the value of the contested variable ending up considerably closer to Agent 1's reference than to Agent 3's, and the contested variable stays at virtually the same value for the remainder of the simulation. Even after this equilibrium point has been reached, and Agent 1, the higher-gain agent, has seemingly gotten the better of the contest, their outputs continue to diverge, thus continuing to intensify the conflict between them, with both agents still striving to bring the contested variable more nearly into line with their own preferred positions. After the contested variable has reached its equilibrium point, each increase in the output of one agent is matched by a nearly equal increase in the output of the other, and the value of the contested variable remains almost unchanged, so that, in terms of "facts on the ground," the conflict has reached a stalemate, even though the intensity of the conflict continues to escalate. Neither party can relax, because each side still perceives a gap between its aspirations and the current situation. Moreover, the contest is still precarious at every moment, because if either side were to stop the escalation unilaterally, its position relative to its own goal would begin to erode.

In sum, a lot is happening in this simple simulation. Winning and losing occur, in addition to the stalemate. Agent 1 emerges the winner, because it does much better than Agent 3 in reaching its goal of closing the gap between the position of the environmental variable and its reference value. By the same token, Agent 3 ends up a clear loser, far less successful in reaching its goal. The stalemate that ensues favors Agent 1, because that agent has more nearly succeeded in bringing the shared environment into line with line with its preferences. Finally, from the point of view of an outside observer, this interaction looks exactly like a positive feedback loop, as escalation by one side provokes counter-escalation by the other and the intensity of the conflict continues to grow.

Although this positive-feedback pattern is exactly what Collins (2012) predicts with his model of conflict, my simulation demonstrates that a PCT model can generate this pattern of escalation without needing a mathematical specification that actually involves positive feedback. In fact, the positive-feedback pattern emerges directly from the conflictive interaction between the two opponents. The other variables that Collins includes as mediating variables in his positive feedback loops, such as group solidarity, atrocities, and ideological polarization, appear not to be necessary for producing escalation of conflict within the PCT model.

Although the additional variables in Collins's conflict model do not appear to be necessary in the PCT model for producing a positive-feedback pattern of conflict, these variables were developed from his own extensive investigations into incidents of face-to-face conflict (2008). Thus, his contention that these variables make a difference in conflict interactions has strong empirical support. I turn next, then, to exploring PCT analogues of the other variables in Collins's model.

Solidarity

The solidarity variable is also central to Collin's model, but the *solidarity* variable is perhaps the least clearly defined of the model's core variables, and this conceptual ambiguity makes finding an analog for the variable within the PCT model less than straightforward. As I noted earlier, solidarity can connote either unity of purpose or else, as Collins puts it, “willingness to sacrifice” (2012:2). In the context of the PCT model, when agents on one side of a conflict all share the same or closely similar reference signals, we can regard them as displaying unity of purpose, and I will take this as my operational definition of solidarity. For an alternative definition of solidarity, willingness to sacrifice might be indexed by the extent to which the agents involved on one side of a conflict all have high loop gain, and therefore devote their attention and energy to the conflict, rather than pursuing other goals.

My previous simulation involved only two agents, one on each side of the conflict. To illustrate group solidarity, however, one must have a group, or at least more than one participant on each side. However, because of the mathematics of the PCT model, it is possible to argue that an agent modeled as a single control system can serve to represent either an individual or a collective actor. In terms of the control exerted on an environmental variable, one high-gain system working alone can have exactly the same effect on a contested variable as several lower-gain systems working together. The three panels of Figure 5 illustrate this principle.

In panel 5A, we see the individual behavior of Agent 1 (defined as in the previous simulations), when it faces no opposition, because the loop gain of the opposing agent has been set to zero. Encountering the initial gap of 20 points between the condition of the contested variable and its own preference, Agent 1 moves rapidly to correct its perceptual error and bring the variable into line with its reference value. As the value of the variable nears the goal of 20 points on the scale, the speed of its error correction slows down, until the variable comes to a stable value that almost, but not quite, reaches 20 points. Specifically, by iteration 100 in this simulation, the contested variable has been brought a value of 19.51176, with each successive iteration bringing the value closer to the goal of 20 points by a tiny increment in the fifth decimal place. Because Agent 1 has been assigned the relatively high loop gain of 40, the control system can do a good job of bringing the controlled environmental variable to its reference value, but control systems can never eliminate error entirely. Nevertheless, within certain limits of stability, the higher the loop gain, the more nearly a control system can succeed in reaching its goal.

[Figure 5A about here.]

Given the mathematics of the PCT model, when multiple control systems use different reference values in attempting to control the same variable, their joint efforts will succeed in stabilizing the variable in spite of the ensuing conflict between them; the equilibrium point we saw in Figure 4 illustrates this stability of a contested variable despite conflict. The equilibrium position emerging from this collective control, however, was a compromise value based on an average of the reference values of the participating systems, weighted by their loop gains. Comparison of Figures 4 and 5A can further illustrate this point. In Figure 5A, as was just mentioned, Agent 1 brings the environmental variable into control at a value of approximately 19.51, just shy of the reference value of 20 points. In Figure 4, in which Agent 1 must contend with Agent 3 for control of the contested variable, the equilibrium point reached is 11.76. Although Agent 1 with its higher loop gain has done a better job of approximating its goal of 20 points than Agent 3, with its goal of –20 points, the compromise value that they reach falls far short of the value that Agent 1 can attain when possessing unfettered control (as in Figure 5A). Thus, the compromises emerging from conflictive interactions satisfy none of the participants.

Panel 5B illustrates the unsatisfactory nature of compromises in another way. In this simulation, all four of the agents in the model have been enlisted to work together. Each agent has been given a loop gain of 10, and Agents 1 to 4 have been assigned reference values of 30, 25, 15, and 10, respectively, a set of values that averages to 20 points. Although the agents all have positive reference values and thus all ostensibly are working together, we see in Figure 5B that Agents 1 and 2 appear to be in conflict with Agents 3 and 4, the two agents with the lowest reference values. As soon as the value of the contested variable goes above their own reference values, those agents start pulling in the negative direction. One of the apparent paradoxes of the PCT analysis is that agents cooperating on a common task are predicted nevertheless to come into conflict, unless all the reference values are precisely identical (see McClelland 2004, 2006). Consideration of how frequently in ordinary situations individuals who attempt to work together come into conflict, and how difficult it can be to get everyone in a group of people on the same page, suggests that this seeming paradox can emerge in everyday interactions, as well.

[Figure 5B about here.]

The most important thing to notice about Figure 5B is that the curve for the value of the contested variable is exactly the same as it was in Figure 5A. The four agents with an average (loop gain-weighted) reference value of 20 have the same impact on the environmental variable as a single agent with that reference value and a combined loop gain equal to the sum of the four. Thus, conflict does not preclude stability of outcome.

Figure 5C demonstrates the effects of collective control in yet another way. As in 5B, all four agents are working together, this time with perfect cooperation. All four have been given the same reference value of 20, and once again the curve for their control of the contested variable is identical to those in panels 5A and 5B. However, in this case the outputs of all four agents coincide at level that is markedly lower than the output of the single agent in Figure 5A. This graph clearly demonstrates the benefits of cooperation in the sense that when the four agents are working together each one has to contribute only one-fourth as much output as would be necessary for a single agent working on its own to reach the same goal.

[Figure 5C about here.]

Returning to the question of how to represent solidarity within the PCT model, it seems reasonable to argue that panel 5C shows an instance of high group solidarity, since their reference values for control are all identical, even though none of the agents has a particularly high loop gain. If all four agents had been given the higher loop gain that Agent 1 had been assigned in panel 5A, they would be even more effective in working together to bring the contested value up to their agreed-on reference value and then in holding it close to that value; in such a case, one might say that solidarity had increased. Thus, solidarity in the PCT model seems to be related both to agreement in reference values and to total loop gain contributed to the joint action. Looking back at panel 5B, we see that if all four agents were to come together on their most extreme reference value, 30, their solidarity, as well as their impact on the contested variable, would clearly have increased. By the same token, if Agent 1, with the highest reference value, had been assigned higher loop gain and nothing else had been changed, their performance as a group in raising the level of the contested variable would have improved, but without any obvious improvement in solidarity (unless solidarity is defined in terms of the loop gain that actors contribute to the joint action). And if all four agents had adopted the reference value of 10, the lowest value in their group, their solidarity as a group would have increased, but with a decrease in their collective impact on the contested variable.

My conclusion is that, in terms of the PCT model, solidarity has no clear effect on conflict intensity or on the effectiveness of a group in pursuing its goals. In any group with scattered reference points for the contested variable, solidarity improves effectiveness in conflict only when the group comes together by agreeing on a more extreme reference value than the current group average. When those with more extreme reference values are able to increase the loop gain that they contribute to the group effort, effectiveness will increase without any clear increase in solidarity. If a group achieves greater solidarity by coming together at a less extreme position, their effectiveness as a unit in conflict will diminish, although the good news is that their internal conflicts will also diminish at the same time.

This analysis suggests that Collins's model of a positive feedback loop connecting solidarity with conflict may be oversimplified. Much depends on the kind of solidarity achieved. Bringing members of a group into agreement will do nothing to escalate a conflict if they coalesce around a shared position that is less extreme instead of more extreme. It appears in this theoretical framework that the effects of solidarity depend on the level of polarization, and thus we turn next to the effects of the polarization variable.

Polarization

In exploring Collins's core variable of *ideological polarization*, I must concentrate on a simplified version of his variable, since the concept of an ideology is far too complex to be captured by my rudimentary model. However, the basic idea of polarization has a straightforward PCT analog: polarization can be indexed by the distance between the reference conditions sought by the opposing parties in a conflict. Simulations with the PCT model show that, when two control systems try to control the same contested variable, the greater the gap between their reference values, the more intense their conflict, if we take intensity to be measured by the divergence of their outputs. Figure 6 illustrates this principle by showing a two-party conflict between the same two agents as were shown in Figure 4, but this time with a smaller gap between their reference values, and thus a reduced level of polarization. In Figure 6, Agents 1 and 3 have reference values of 10 and –10, respectively, instead of 20 and –20. The vertical scale of the graph in Figure 6 remains the same as in Figure 4, and by comparing Figure 6 to Figure 4 one can easily see the reduction in the divergence of the output curves, and thus the intensity of the conflict. It is worth noting, however, that if the loop gains of the agents are high enough even small differences in reference values between opponents can lead to large differences in their outputs (see McClelland 2004).

[Figure 6 about here.]

The conclusion to be drawn from these PCT simulations is that polarization between the parties of a conflict contributes to its intensity by increasing the rate of escalation and counter-escalation, a conclusion that agrees well with Collins's theory, when he puts the *ideological polarization* variable into one of the positive feedback loops in core of his model (see Figure 1). Nevertheless, my examination of the effects of polarization, in conjunction with my simulations of solidarity (Figures 5A, B, and C), suggests that some rearrangement of the variables in his core model might be in order. Solidarity, as we have seen, works best to increase the group's effectiveness in conflict when its members coalesce around more extreme positions. Solidarity without polarization has little effect. The model presented by Collins puts the polarization and solidarity variables into separate feedback loops (see Figure 1), but my analysis suggests that the two factors might more properly be seen as working together in the same feedback loop.

Atrocities

The final core variable in Collins's model, *atrocities,* also poses significant challenges for translation into PCT terms. In his words, atrocities are "opponents' actions that we perceive as especially hurtful and evil, a combination of physical and moral offense that we find outrageous" (2002:2). These complicated perceptions are obviously far beyond the capability of my simple PCT agents to simulate. However, Collins has put the atrocities variable into the same positive feedback loop as the polarization variable. Atrocities, according to Collins, increase the polarization of members of a group, who take more extreme positions because they feel themselves to have been victimized, and the effects of such polarization on the intensity of the conflict are easy to model with PCT.

Figure 7 presents a scenario in which tit-for-tat atrocities by opposing sides in a conflict increases the polarization in reference values between the two sides and leads to more intensified conflict. For simplicity in presentation, the simulation is restricted to only two actors, Agents 1 and 3, just as in the simulations shown in Figures 4 and 6. The substantive conclusions would be the same, however, for a simulation in which all four modeled agents were active.

[Figure 7 about here.]

In Figure 7 the simulation is based on the assumption that unspecified atrocities take place at regular intervals, every 20 iterations, first on the part of one side and then the other. While the perceptions of atrocity are not modeled in the simulation, the hypothesized effects of the atrocity in terms of increased polarization are shown. Each instance of an atrocity is followed by an increase (in absolute value) of 10 points in the reference signal for the agent supposed to have perceived the atrocity. The first atrocity, then, takes place at iteration 20, and the reference value for Agent 3, representing the victimized side, jumps from –20 to –30. Atrocity number two, at iteration 40, results in a jump in the reference value for Agent 1, from 20 to 30, and so forth. After two more exchanges of atrocities, the reference values of Agents 1 and 3 end the simulation at 40 and –40, respectively.

As expected, the pace of intensification of the conflict in Figure 7 increases after each of the designated atrocities. These increases are easiest to discern in the output of Agent

1, modeled to have a loop gain of 40, in comparison to Agent 3's loop gain of 10. The effects of the jumps in polarization can also be seen in the curve for the contested variable. From iterations 1 to 20, the efforts of Agent 1 prevail over those of Agent 3 and bring the contested variable almost to the equilibrium point that was achieved in the simulation shown in Figure 4. From iteration 21 to 40, the jump to a more polarized reference value by Agent 3 slightly increases its output, which means that the value of the contested variable begins to decline, only to start moving upward again toward a higher equilibrium point when Agent 1 jumps to its more polarized reference value at iteration 41. The same patterns repeat at iterations 61 and 81. If atrocities increase polarization, the greater the polarization, the more intense the conflict.

An alternative theoretical possibility is that atrocities could be represented in a PCT model as leading to increases not in polarization, but in loop gain. The idea here is that, having an emotional response to a perceived atrocity, members of the victimized side would redouble their efforts to win the contest, without necessarily changing their goals of what winning would mean. Thus, each atrocity would be followed by a jump in the loop gain that actors are applying to their control loops. When put to empirical test, the outcome of this scenario would be hard to distinguish from the outcome shown in Figure 7. A simulation (not shown) in which increases in loop gain were substituted for jumps in polarization following each hypothesized atrocities produced patterns of increasing intensity in conflict and changing values of the contested variable that were only subtly different from those shown in Figure 7. Since neither the reference values nor loop gains would be directly visible to outside observers, a researcher investigating empirical instances of conflict would have difficulty distinguishing among these theoretical possibilities: polarization, or loop gain, or perhaps both.

DISCUSSION

As I said at the beginning, the new theory of conflict escalation and de-escalation offered by Collins (2012) represents an impressive achievement. His theory builds on substantial empirical support, challenges sociologists to take the dynamics of time seriously in their theoretical models, and points to a fruitful approach for dealing with time dynamics by invoking systems theory, with its mathematical base. In this paper, I have used a systems approach to reformulate Collins's theory as a multi-agent mathematical model that is capable of reproducing some of his theory's predicted outcomes.

What has been learned from this exercise of translating into mathematical terms a theory originally cast in the familiar form of verbal statements and schematic diagrams? First, the discipline of attempting to construct mathematical specifications for key variables in the theoretical framework has revealed conceptual ambiguities in the verbal formulations. The central variable in the model, conflict, appears to be better conceptualized as an interactional outcome of "action at cross purposes" (Collins 2009:569) than as a variable in itself, although the intensity of conflict was possible to model as a mathematical variable. The traditional definition of solidarity, another key variable in his conceptual framework, is open to a variety of interpretations, with only one of the interpretations—solidarity as agreement in reference signals—easily amenable to mathematical specification. Similarly, the definition of polarization appears to be ambiguous, referring either to a divergence in reference values between two sides of a conflict or to one side's position along an ideological continuum. Again, the exigencies of needing a precise mathematical definition required that only one of those options be chosen. Obviously, mathematical specification forces greater conceptual clarity than verbal statements may allow, at the cost of some richness of evocation.

Second, the simulation results have revealed patterns of dynamic interaction not immediately apparent from the verbal formulations. Most strikingly, the expected pattern of positive feedback in conflict escalation was shown to emerge directly from the interactions of agents modeled as negative-feedback control systems. Simulations also showed that conflicts in which combatants can increase their outputs without any limit tend to move toward a stalemate in which the two sides continue escalating their efforts with little effect on the contested variable. Other simulations showed how an increase in solidarity, when defined as agreement in reference values among combatants on one side of a conflict, can actually diminish the combative effectiveness of the group as a whole, if members of the group don't agree on more extreme reference values for the contested variable. Furthermore, if group solidarity (defined as agreement) is low, a simulation showed how partial success in moving toward a group's goal may lead to internal conflict within the group, as those who are satisfied with a partial victory start to pull back from further escalation of the conflict. Finally, simulations involving the polarization variable—defined as divergence in reference values between two sides of a conflict—suggested that the schematic diagrams presented by Collins might benefit from some rearrangement of the variables, for example, by inserting polarization into the same feedback loop as solidarity, since solidarity in the absence of polarization appears to have little effect. Thus, the results of mathematical simulations have suggested a variety of elaborations and modifications of the theory that might have been harder to see from its verbal formulation.

A third benefit of reformulating this theory of conflict escalation as a multi-agent mathematical model is that it increases the scope of possible applications of the theory. Collins focused his theory on explaining the escalation of physically violent conflicts, but the mathematical model that I have presented, with its greater level of abstraction, can be applied to many other types of interactions in which conflicts develop as people attempt to control variables in their shared environments using disparate reference standards. Political controversies, for instance, are subject to similar kinds of escalations and stalemates as wars and other violent controversies, and so are family feuds. Even everyday interactions in which conflicts develop in the midst of ostensibly cooperative projects can be described by the same set of modeling tools (for an example, see McClelland 2006). By moving to a more abstract mathematical formulation, the generality of a theory can be enhanced.

Where might investigators go next in developing this control-theory mathematical approach to theories of conflict? One possibility is to extend these simulations to the other variables in Collins's theoretical framework. A second possibility is to test the simulations against empirical data on actual conflicts, though an investigator seeking to go this route will need access to much better real-time data on the moment-to-moment development of conflicts than has generally been available.[5] A third and yet more challenging possibility is to begin constructing more elaborate mathematical models of conflict processes that incorporate multiple levels of perception instead of focusing on a single contested variable.

This paper is one of several recent theoretical and substantive studies that apply the mathematical tools of systems analysis to the dynamics of conflict (Bartoli, Bui-Wrzosinska and Nowak 2010; Coleman *et al.* 2007; Fletcher *et al.* 2011; Liebovitch, Vallacher and Michaels 2010; Musallam, Coleman and Nowak 2010; Nowak *et al.* 2010; Praszkier, Nowak and Coleman 2010), in what appears to be a minor resurgence of systems thinking in this substantive area. Among these recent analyses of conflict, however, my PCT analysis has been the only one to link the dynamics of conflict to processes of control. Although control is, arguably, the "central notion" of sociology (Gibbs 1989), sociologists have been slow to focus on control processes as mechanisms of conflict. Obviously, when a person or group of people attempt to exert power over others by controlling their actions, conflict often ensues, and conflicts regularly emerge over control of resources. But the idea that struggles over control are central to all kinds of social conflict has rarely been articulated in the sociological literature. If my mathematical model helps to underline the importance of looking at the perceptions people are trying to control as we seek to understand the dynamics of their interactions, it will have made a useful contribution to the development of theory in this area.

NOTES

1. Pierre Bourdieu is one notable exception to the generalization that sociologists have neglected the time dimension in their analyses. In his field theory, Bourdieu (1990:66-67) uses the analogy of a football or tennis player to describe the way

that an individual interacting in a social field must adjust to moment-by-moment changes in his environment. Despite his theoretical emphasis on the importance of time, Bourdieu's empirical work (*e.g.,* 1984), like most sociological analyses, does not focus on close analysis of the time dynamics of specific incidents.

2. Systems thinking emerged in the first half of the 20^{th} century, as American engineers worked to stabilize the newly developing technology of long-distance telephone service, and investigators supported by the armed forces sought to improve systems for aiming guns in naval warfare and shooting down enemy aircraft (see Mindel 2002). Mathematician Norbert Wiener, who was involved in war research himself, helped to lay the mathematical foundation for the understanding of negative feedback control systems and gave the name "cybernetics" to this emerging field (Wiener 1948). Cybernetics enjoyed a burst of popularity in the middle of the 20^{th} century, influencing such prominent sociologists as Talcott Parsons, but then faded for a variety of reasons, including mistakes made by Wiener and other advocates in presenting the approach, the emergence of a competing perspectives like artificial intelligence research based on analogies between the brain and the newly developing digital computer, and the drying up federal funding for cybernetics research in the Cold War after Russian scientists embraced Wiener's work enthusiastically (see Conway and Siegelman 2005, Hayles 1999).
3. Fletcher et al. (2011) performed mathematical simulations based on a theory of battle dynamics proposed by Collins (2010), but their approach was substantially

different from the multi-agent modeling that I use in this paper. Another systems-modeling approach to the analysis of conflict makes use of "dynamical social psychology" (*e.g.,* Liebovitch, Vallacher and Michaels 2010; Nowak *et al.* 2010; see also Vallacher and Nowak 1997).

4. Both the loop gains and the slowing factor for these simulations have been selected arbitrarily, but when loop gain is sufficiently high (greater than 10) control systems perform in a predictably stable manner over a wide range of loop-gain values. Had I run these simulations with other combinations of loop-gain and slowing-factor values (within a reasonable range), it would not have changed any of the substantive results.
5. In a pioneering example of this approach, Heise (2006) has applied Affect Control Theory, a dynamic model related to PCT, to the analysis of data from diplomatic exchanges between Israel and its Arab neighbors.

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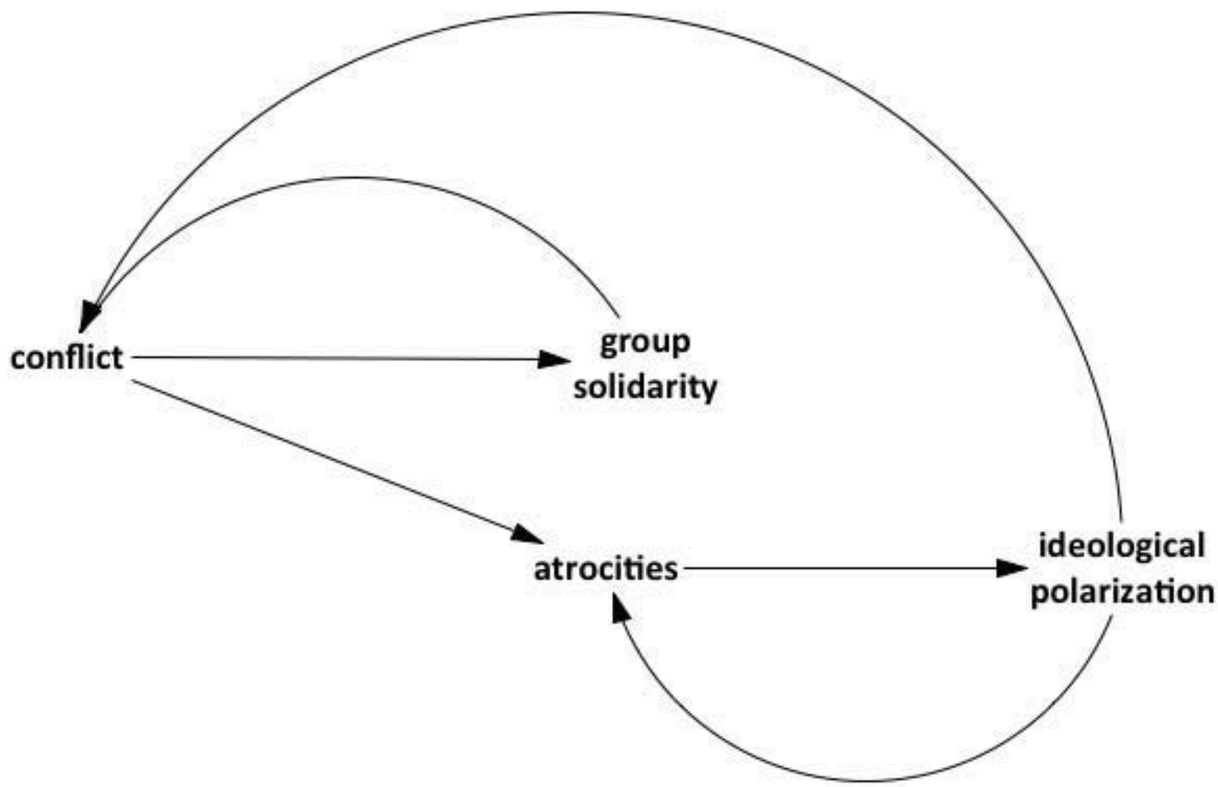

Figure 1. Positive Feedback Loops in Collins's Theory of Conflict Escalation

(Reproduction of Collins [2012] Figure 3, p. 4).

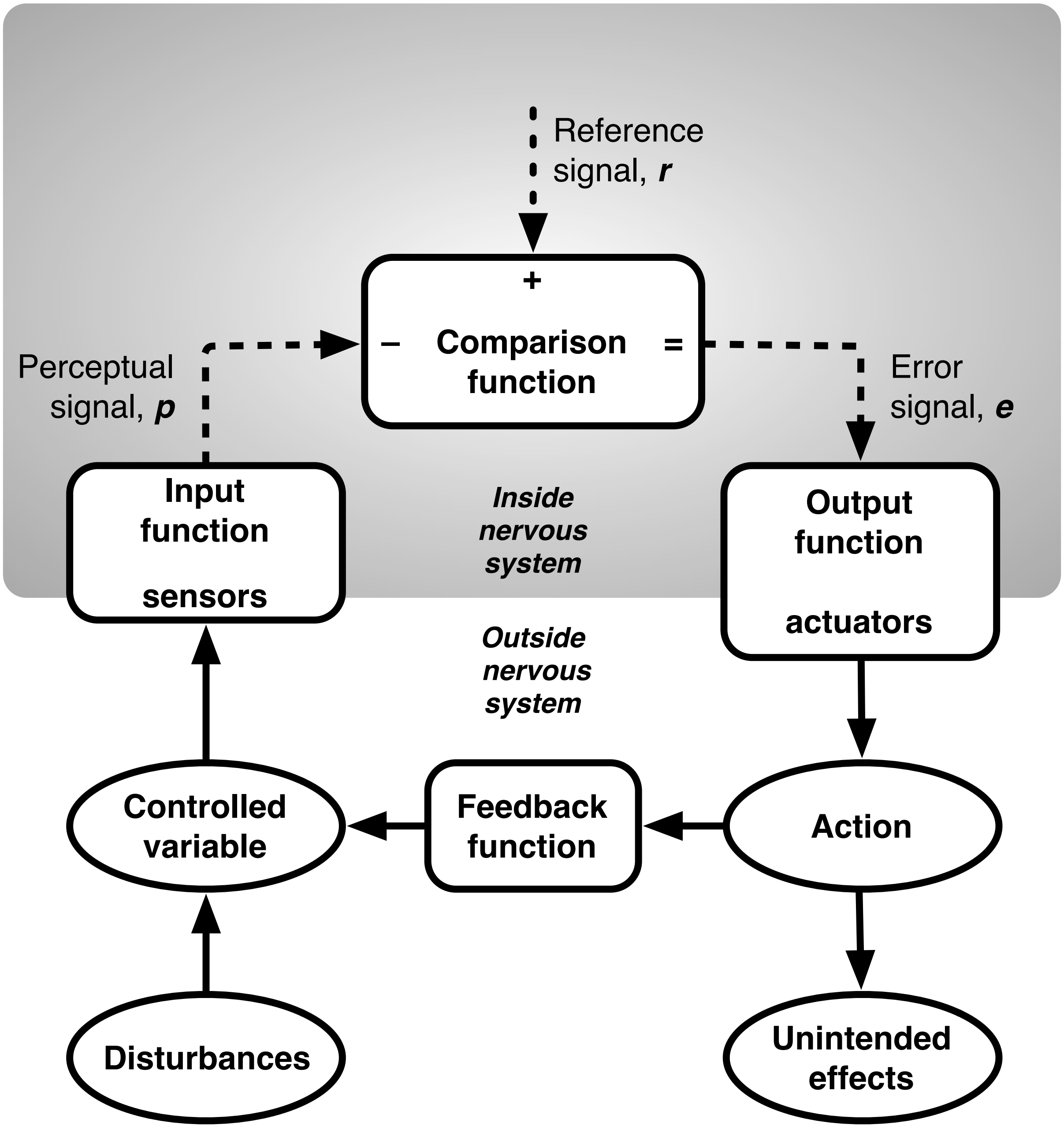

Figure 2. The Perceptual Control Theory Model of a Negative Feedback Loop

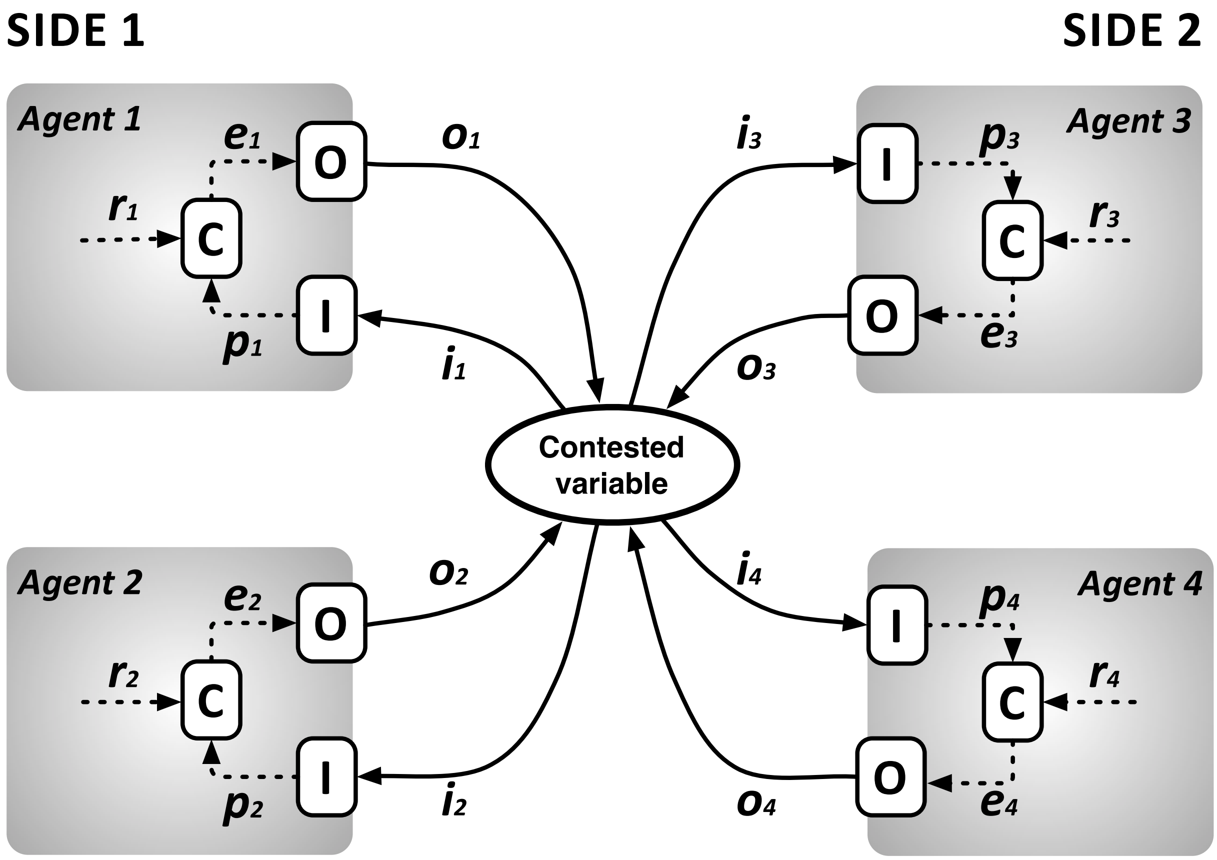

Figure 3. The Simulation Model Used in this Study

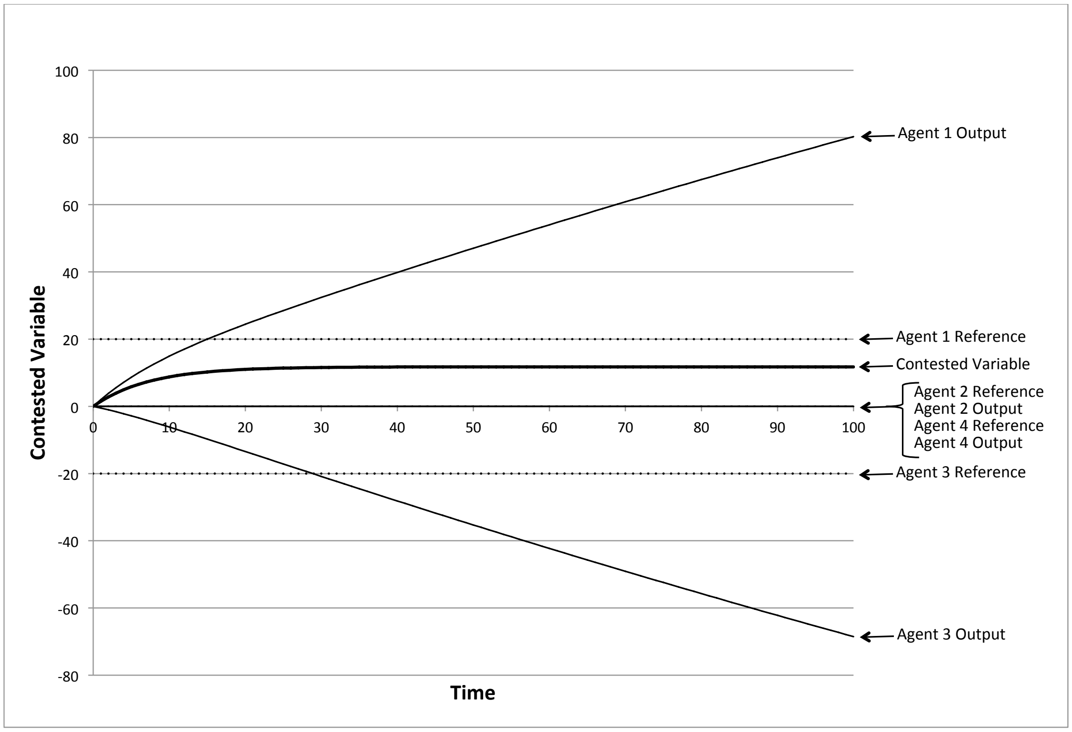

Figure 4. Simulation of Two-Party Conflict

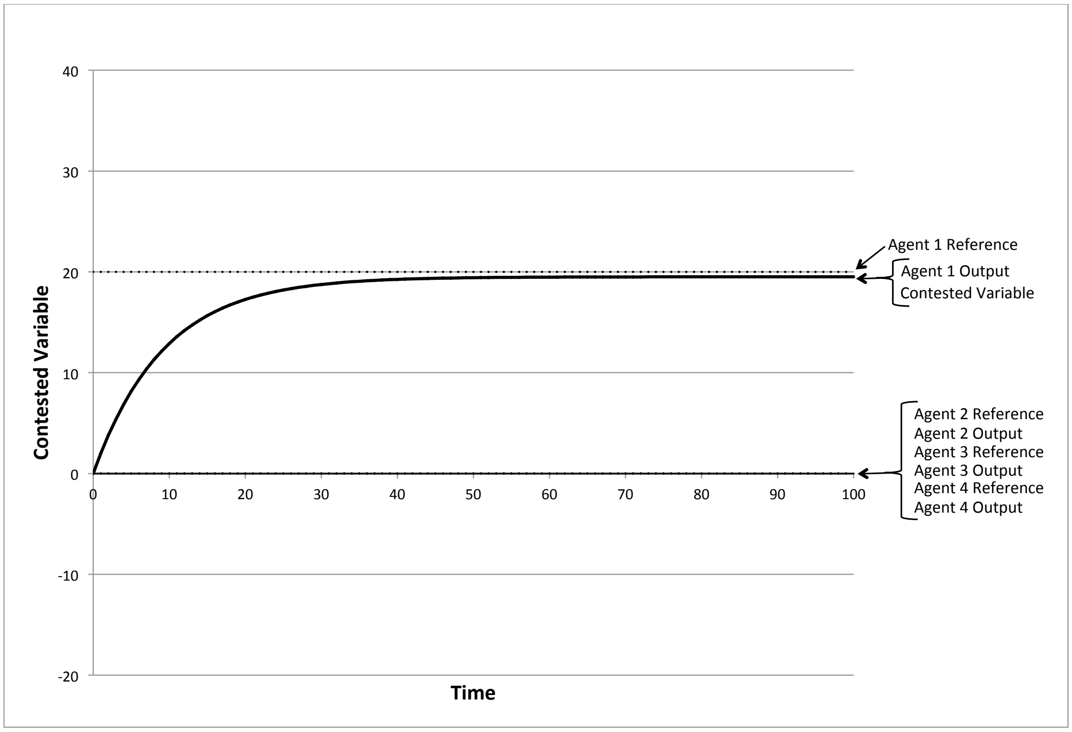

Figure 5A. Control by a Single Agent with Loop Gain = 40

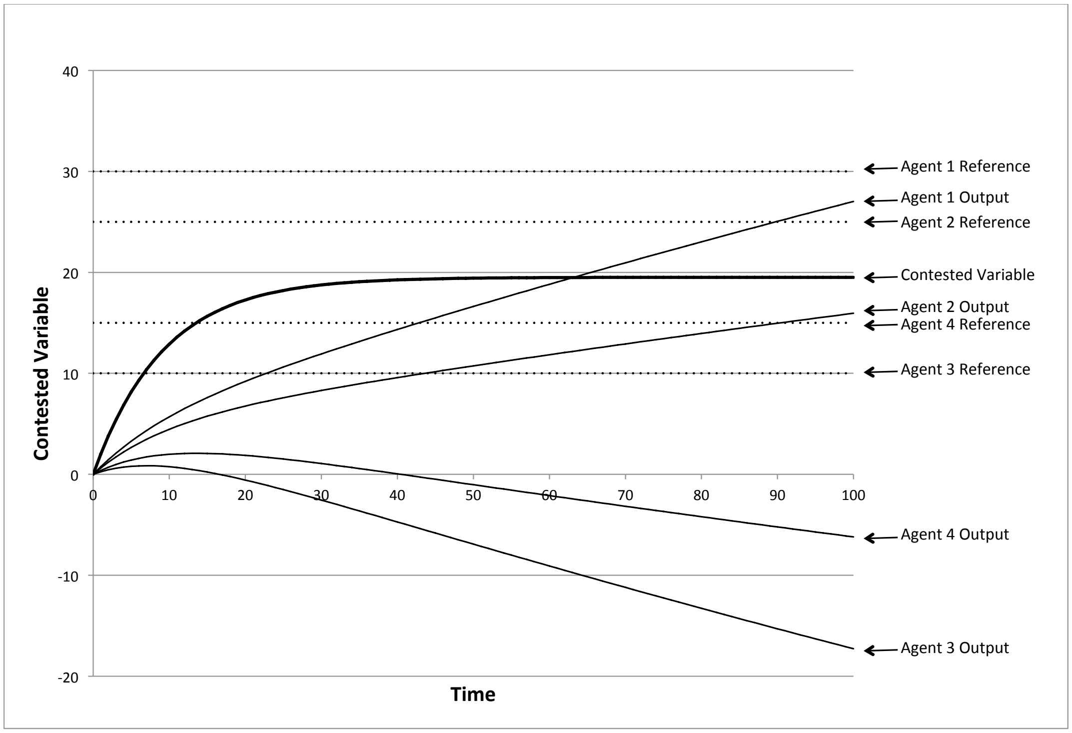

Figure 5B. Collective Control by Four Agents, Each with Loop Gain = 10

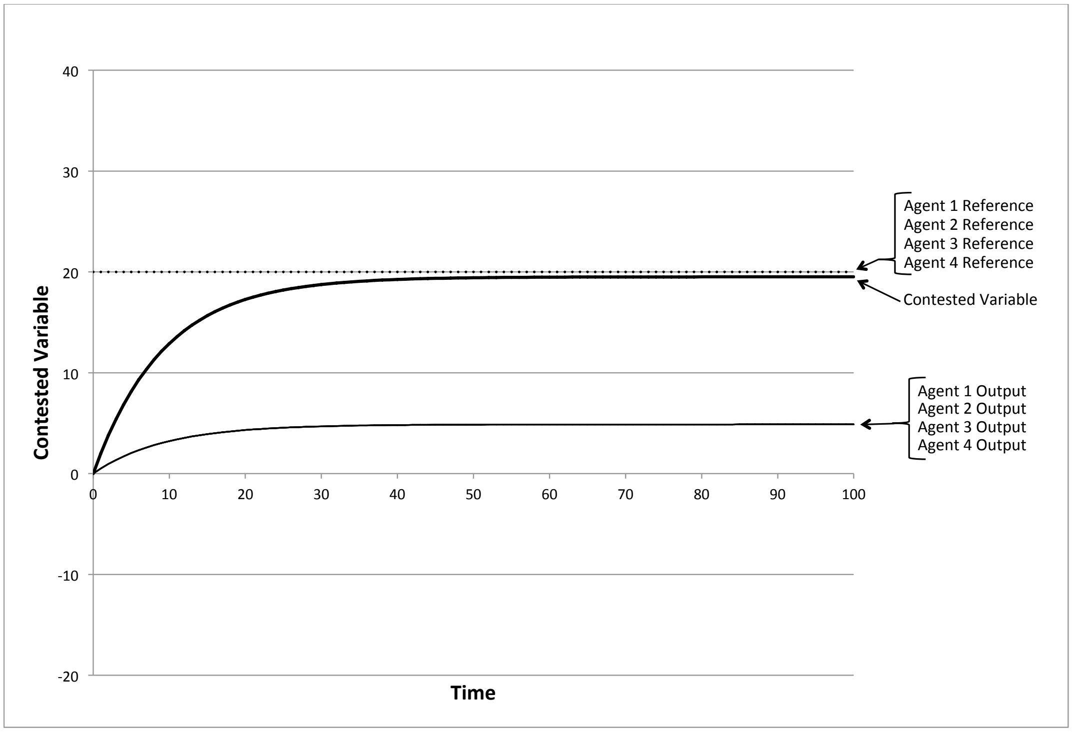

Figure 5C. Collective Control by Four Agents with Identical Reference Values and Loop Gain =10

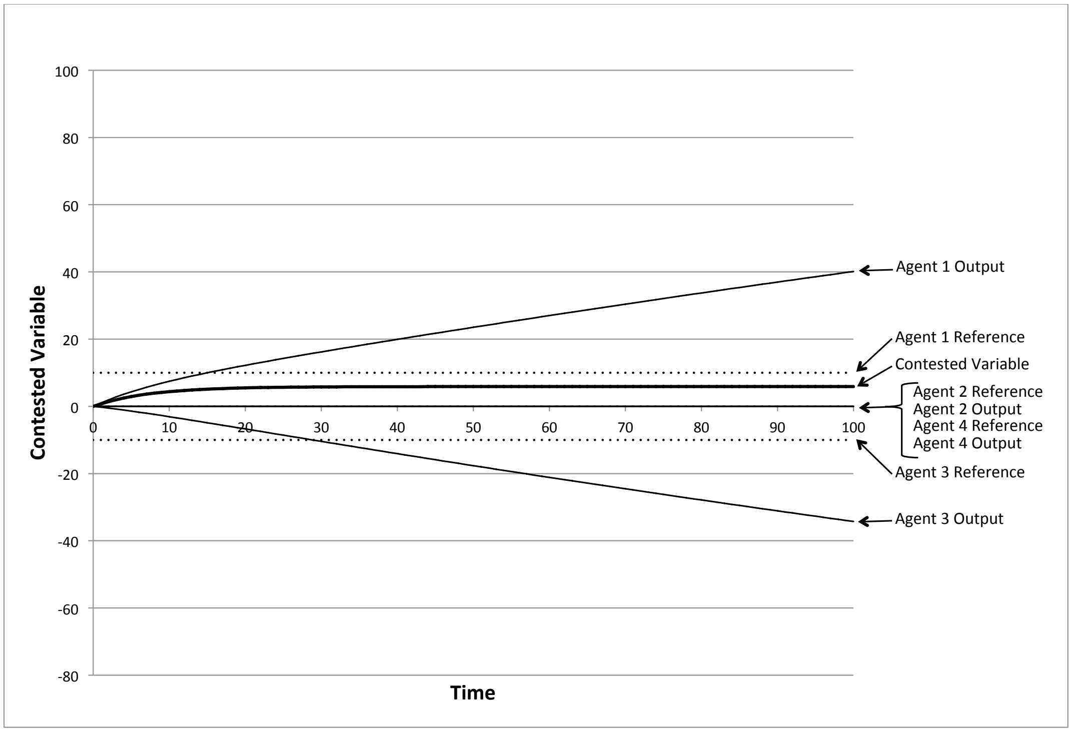

Figure 6. Simulation of Two-Party Conflict with Reduced Polarization

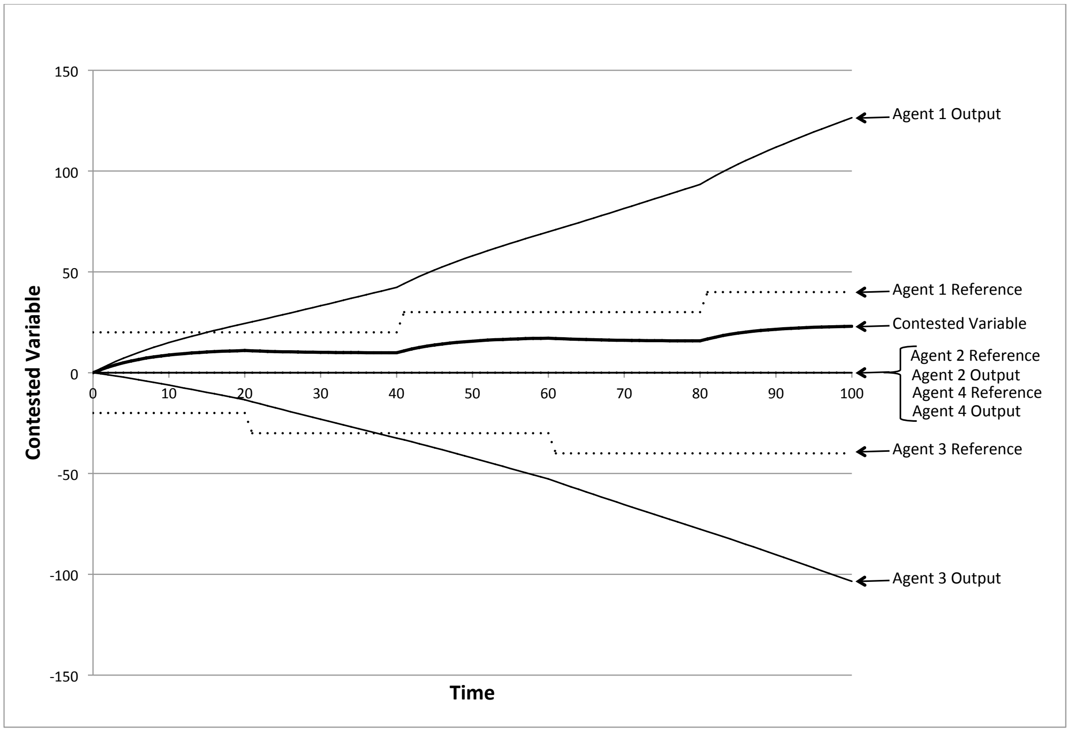

Figure 7. Simulation with Atrocities Represented as Increases in Polarization