Regression Coefficient of y on x:

$$b_{yx} = r_{yx}\frac{s_y}{s_x}, \quad \text{or} \quad r_{yx}\frac{SD_y}{SD_x} \tag{5.27}$$

Regression Coefficient of x on y:

$$b_{xy} = r_{xy}\frac{s_x}{s_y}, \quad \text{or} \quad r_{xy}\frac{SD_x}{SD_y} \tag{5.28}$$

When the original measures of a bivariate distribution have been transformed to z deviates, and consequently the two variables have equal standard deviations, the regression coefficients are equal to r and hence to each other:

$$b_{z_y z_x} = r_{yx} \tag{5.29}$$

$$b_{z_x z_y} = r_{xy} \tag{5.30}$$

Under these conditions, r measures the slope of the regression lines and is thus the tangent of the angle a in Fig. 5.6 for the regression of z_y on z_x. and is the tangent of the angle for the regression of z_x on z_y. Hence the regression equation of z_y on z_x simplifies to:

$$\tilde{z}_y = r_{yx}z_x \tag{5.31}$$

and that for z_x on z_y is:

$$\tilde{z}_x = r_{xy}z_y \tag{5.32}$$

A trigonometric way to determine the value of r, therefore, consists in (1) setting up equated scales (by z scores) for a bivariate distribution, as in Fig. 5.7; (2) plotting the average variation of z_y with respect to successive interval values of z_x; (3) drawing a straight line to these averages so that the errors of fit will be as small as possible; and (4) then determining the value of the tangent of angle a. As a check, the average variations of z_x with respect to z_y can also be plotted, the line fitted, and the tangent of angle a' determined.

It is to be noted that the order of the x and y subscripts for r may be either xy or yx; consequently, when the r referred to is clear by context, the subscripts are usually omitted. On the other hand, since $b_{yx} \neq b_{xy}$